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Semi-Analytic Solution of Non-Linear Coupled Differential Equation using Adomian Decomposition

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ABSTRACT [ENGLISH/ANGLAIS]

In this paper, Numerical algorithm is adopted to solve strong coupled nonlinear system of Ordinary Differential Equations. The research work aimed at obtaining series solutions to boundary valued problems. This method shows an accurate and efficient technique in comparison with numerical solutions. Adomian Decomposition method provides highly precise semi analytical solution for strong nonlinear coupled differential equations.

Keywords: Adomian decomposition method, bvp4c, coupled ODE, numerical methods

RÉSUMÉ [FRANÇAIS/FRENCH]

Dans cet article, l'algorithme numérique est adopté pour résoudre un solide système non linéaire ordinairement couplé à des équations différentielles. Les travaux de recherche visent à obtenir des solutions de la série à la valeur limite des problèmes. Cette méthode montre une technique précise et efficace en comparaison avec les solutions numériques. La méthode de décomposition Adomian fournit une solution semi analytique hautement précise pour les solides, non linéaires, équations différentielles couplées.

Mots-clés: Méthode de décomposition Adomian, bvp4c, ODE Couplé, méthodes numériques

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INTRODUCTION

Adomian decomposition method was introduced by G. Adomian (1923-1996) and is one of a semi analytical method for solving approximate (series) solutions for large classes of nonlinear and linear differential equations. Recently, classical ADM has been reported to be a very good method for solving strong nonlinear mathematical models. Since then, this method has been known as the Adomian Decomposition method [1, 2]. The main goal of this method is towards a unified theory for the solution of linear and nonlinear, ordinary or partial differential equations either initial and boundary value problems. The technique is based on a decomposition of a solution of a nonlinear operator equation in a series of functions. Al-Khaled and Allan [3] implemented the Adomian method for variable-depth shallow water equations with a source term and illustrated the convergence numerically. A comparative study between the ADM and the Sinc-Galerkian method for solving population growth models was performed by Al-Khaled [4]. Shawagfeh and Kaya [5] carried out a comparative analysis between ADM and Runge Kutta method for solving system of ordinary differential equations. Wazwaz [6] developed a numerical algorithm to approximate solution of higher-order boundary value problems. Application of Chebyshev polynomials to numerical implementation of the Adomian decomposition method were discussed by Hosseini [7]. ADM has successfully been applied to many situations, accuracy of ADM was applied to Lorenz system. Hashim et al. [8], and Saha and Bera [9] used Adomian Decomposition Method to give approximation solution to extraordinary differential equation, and Dogan [10] solved generalized nonlinear Boussinesq equations. Afrouzi [11] applied ADM to solve reaction diffusion equation, [12] solved Riccati differential using ADM and compared the numerical result with exact solution. Adomian and Rach

[13] used ADM to obtain the solution of algebraic equations of quadratic, cubic and general higher order polynomial equations and negative or non-integral powers and random algebraic equations. Awangkechil and Hashim [14] were among the first researchers to apply ADM to solve 2 by 2 system of weak nonlinear ordinary differential equations which was derived from free convective boundary layer equation. Solution of problems with boundary conditions involving a limit at infinity has been a very difficult problem in solving boundary value problems. In this work, bvp4c package on MATLAB is used to determine the values at infinity. Adomian Decomposition method is adopted to find closed form semi analytical solutions to the first example.

ADOMIAN DECOMPOSITION METHOD

Considering a differential equation of the form

$$Lu + Ru + Nu = g(x)$$

where L is the linear operator which is the highest order derivative, Ru is the remainder of linear operator including derivatives of less order than L, Nu represents the nonlinear terms and g is the source term.

Applying the inverse operator L^{-1} and rearrangement

$$u = L^{-1}g(x) - L^{-1}R(u) - L^{-1}N(u)$$

L^{-1} represent the inverted highest order operator L.

For Example, if L is the third derivatives $L = \frac{d^3}{d\eta^3}$, then L^{-1} is a three folds integration operator $L^{-1} = \int_0^\eta\int_0^\eta\int_0^\eta (*)d\eta d\eta d\eta$. After integrating the source term and combining it with the term and combination with the term arising from given initial and boundary conditions, a function $f(x) = L^{-1}g(x)$ is introduced

$$u = f(x) - L^{-1}R(u) - L^{-1}N(u)$$

The first three terms can be written as

$$u = \sum_{n=0}^{\infty} u_n$$

$$u_0 = f(x)$$

$$u_1 = -L^{-1}(Ru_0) - L^{-1}(Nu_0)$$

$$u_2 = -L^{-1}(Ru_1) - L^{-1}(Nu_1)$$

$$u_n = -L^{-1}(Ru_{n-1}) - L^{-1}(Nu_{n-1})$$

In ADM, $Nu = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n)$ and A_n are called Adomian Polynomials and depend only on the u components and make a rapid convergent series. Nonlinearity term is written here in terms of A_n and Nu which need not even be analytic. The Adomian polynomials is generated by

$$A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N \sum_{i=0}^{n} u_i \lambda^i\right]_{\lambda=0}$$

Semi analytical solution for the kth term approximation is

$$u_k = \sum_{i=0}^{k=1} u_k \text{ where } u = \lim_{n\to\infty} u_k$$

1.1 Consider

$$\frac{d^3 f}{d\eta^3} + 3f\frac{d^2 f}{d\eta^2} - 2\frac{df}{d\eta}\frac{df}{d\eta} + \theta = 0 \qquad (1)$$

$$\frac{d^2\theta}{d\eta^2} + 3\Pr f\frac{d\theta}{d\eta} = 0 \qquad (2)$$

Subject to boundary conditions

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 0,$$
$$\theta(0) = 1,\ \theta(\infty) = 0 \qquad (3)$$

To solve the system of coupled ODEs, rearranging (1) and (2) making the highest derivative to remain on RHS and rewrite as

$$f''' = -3ff'' + 2(f')^2 - \theta \qquad (4)$$

$$\theta'' = -3\Pr f\theta' \qquad (5)$$

Adopting the standard procedure of ADM by introducing $L_1 = \frac{d^3}{d\eta^3}$ and $L_1 = \frac{d^3}{d\eta^3}$ with inverse operators $L_1^{-1}(*) = \int_0^\eta\int_0^\eta\int_0^\eta (*)d\eta d\eta d\eta$ and $L_2^{-1}(*) = \int_0^\eta\int_0^\eta (*)d\eta d\eta$ thus, equation (4) and (5) becomes

$$L_1 f = -3ff'' + 2(f')^2 - \theta \qquad (6)$$

$$L_2\theta = -3\Pr f\theta' \qquad (7)$$

Applying the inverse operator on both sides of (6) and (7)

$$L_1^{-1}(L_1 f) = -3L_1^{-1}ff'' + 2L_1^{-1}(f')^2 - L_1^{-1}\theta \qquad (8)$$

$$L_2^{-1}(L_2\theta) = -3\Pr L_2^{-1}f\theta' \qquad (9)$$

Adomian decomposition method assumes that the unknown function $f(\eta)$ and $\theta(\eta)$ can be expressed by an infinite series of the form

$$f(\eta)=\sum_{m=0}^{\infty} f_m = f_0 + f_1 + \ldots + f_\infty \text{ and}$$

$$\theta(\eta)=\sum_{m=0}^{\infty}\theta_m = \theta_0+\theta_1+\ldots+\theta_\infty \tag{10}$$

The linear and nonlinear terms can be decomposed by an infinite series of polynomials given by

$$\sum_{m=0}^{\infty} A_m = ff'', \qquad \sum_{m=0}^{\infty} B_m = (f')^2,$$

$$\sum_{m=0}^{\infty} C_m = \theta, \qquad \sum_{m=0}^{\infty} D_m = f\theta' \tag{11}$$

Where the components $f_m, \theta_m, A_m, B_m, C_m$ and D_m will be determined recurrently. Also, A_m, B_m, C_m and D_m are the Adomian Polynomials which make a rapidly convergent series. The Adomian polynomial is generated by the following formula

$$\frac{1}{n!}\frac{d^m}{d\lambda^m} N\left(\sum_{i=0}^{n} u_i \lambda^i\right)_{\lambda=0} \tag{12}$$

The Exact solution is $f(\eta)=\underset{n\to\infty}{Lim}\sum_{m=0}^{n} f_m$ and

$\theta(\eta)=\lim_{n\to\infty}\sum_{m=0}^{n}\theta_m$. RHS of (8) and (9) becomes

$$L_1^{-1}(L_1 f) = f(\eta) - f(0) - \eta f'(0) - \frac{\eta^2}{2} f''(0)$$

$$L_2^{-1}(L_2\theta)=\theta(\eta)-\theta(0)-\eta\theta'(0)$$

From (3), substitute $f(0)=0,\ f'(0)=0,\ f''(0)=P$ $\theta(0)=1, \theta'(0)=Q$

$$L_1^{-1}(L_1 f) = f(\eta) - \frac{\eta^2}{2}P \tag{13}$$

$$L_2^{-1}(L_2\theta)=\theta(\eta)-1-\eta Q \tag{14}$$

Substitute (13) and (14) back into (8) and (9)

$$f(\eta)=\frac{\eta^2}{2}P - 3L_1^{-1} ff'' + 2L_1^{-1} f'^2 - L_1^{-1}\theta$$

$$\theta(\eta)=1+\eta Q - 3L_2^{-1}\Pr f\theta'$$

The outcome equations can be decomposed down into

$$f_0(\eta)=\frac{\eta^2}{2}P \text{ and } \theta_0(\eta)=1+\eta Q$$

$$f_{m+1}(\eta) = -3L_1^{-1} ff'' + 2L_1^{-1} f'^2 - L_1^{-1}\theta \tag{15}$$

$$\theta_{m+1} = -3L_2^{-1}\Pr f\theta' \tag{16}$$

Next step is to substitute all the Nonlinear and linear terms into (15) and (16)

$$f_{m+1}(\eta) = -3L_1^{-1}\sum_{m=0}^{\infty} A_m + 2L_1^{-1}\sum_{m=0}^{\infty} B_m - L_1^{-1}\sum_{m=0}^{\infty} C_m \tag{17}$$

$$\theta_{m+1} = -3L_2^{-1}\Pr\sum_{m=0}^{\infty} D_m \tag{18}$$

Using Adomian polynomial formula (12) for $m=0,1$ and 2 on the series (17) and (18) Starting with when $m=0$

$$A_0 = f_0 f_0'', \qquad A_1 = f_1 f_0'' + f_0 f_1''$$

$$A_2 = f_2 f_0'' + f_1 f_1'' + f_0 f_2''$$

$$B_0 = f_0'^2,\ B_1 = 2 f_0' f_1' \text{ and } B_2 = 2 f_2' f_0' + f_1'^2$$

$$C_0=\theta_0,\ C_1=\theta_1 \text{ and } C_2=\theta_2$$

$$D_0 = f_0\theta_0',\ D_1 = f_0\theta_1' + f_1\theta_0'$$

$$D_2 = f_0\theta_2' + f_1\theta_1' + f_2\theta_0'$$

When m = 0 equations (17) and (18) becomes

$$f_1(\eta) = -3L_1^{-1}A_0 + 2L_1^{-1}B_0 - L_1^{-1}C_0 \tag{19}$$

$$\theta_1(\eta) = -3\Pr L_2^{-1}D_0 \tag{20}$$

Transforming the polynomials for a case when $m=0$

$$A_0 = f_0\frac{d^2 f}{d\eta^2} = \frac{\eta^2}{2}P^2$$

$$B_0 = \left(\frac{df_0}{d\eta}\right)^2 = \eta^2 P^2$$

$$C_0 = \theta_0 = 1+\eta Q$$

$$D_0 = f_0\frac{d\theta_0}{d\eta} = \frac{\eta^2}{2}PQ$$

Substituting the above polynomials into (19) and (20) and introduce inverse operator

$$f_1(\eta)=\int_0^\eta\int_0^\eta\int_0^\eta(-3A_0+2B_0-C_0)\,d\eta\, d\eta\, d\eta$$

$$\theta_1(\eta)=\int_0^\eta\int_0^\eta(-3\Pr D_0)\,d\eta\, d\eta$$

Then substituting all the polynomials above to yield

$$f_1(\eta)=\frac{1}{120}P^2\eta^5-\frac{1}{6}\eta^3-\frac{1}{24}\eta^4 Q \tag{21}$$

$$\theta_1(\eta) = -\frac{1}{8}\Pr PQ\eta^4 \tag{22}$$

When $m=1$ equations (17) and (18) becomes

$$f_2(\eta) = -2L_1^{-1}A_1 + 2L_1^{-1}B_1 - L_1^{-1}C_1 \tag{23}$$

$$\theta_2(\eta) = -3L_2^{-1}\Pr D_1 \tag{24}$$

Transforming the polynomials

$$A_1 = f_1\frac{d^2 f_0}{d\eta^2} + f_0\frac{d^2 f_1}{d\eta^2} = \frac{11}{120}\eta^5 P^3 - \frac{7}{24}\eta^4 QP - \frac{2}{3}\eta^3 P$$

$$B_1 = 2f_0' f_1' = 2\frac{df_0}{d\eta}\frac{df_1}{d\eta} = \frac{1}{12}\eta^5 P^3 - \frac{1}{3}PQ\eta^4 - P\eta^3$$

$$C_1 = \theta_1 = -\frac{1}{8}\Pr PQ\eta^4$$

$$D_1 = f_0\frac{d\theta_0}{d\eta} + f_1\frac{d\theta_1}{d\eta} = -\frac{1}{4}\eta^5 P^2 \Pr Q + \frac{1}{120}\eta^5 P^2 Q^3 Q - \frac{1}{24}\eta^4 Q^2 - \frac{1}{6}\eta$$

Substituting the above polynomials into (23) and (24) and introduce inverse operator

$$f_2(\eta) = \int_0^\eta\int_0^\eta\int_0^\eta (-3A_1 + 2B_1 - C_1)\, d\eta\, d\eta\, d\eta$$

$$\theta_2(\eta) = \int_0^\eta\int_0^\eta (-3\Pr D_1)\, d\eta\, d\eta$$

Then substituting all the polynomials above to yield

$$f_2(\eta) = -\frac{13}{40320}P^3\eta^8 + \frac{1}{1008}PQ\eta^7 + \frac{1}{1680}\Pr PQ\eta^7 \quad (25)$$

$$\theta_2(\eta) = \frac{1}{56}P^2 Q\eta^7 \Pr^2 - \frac{1}{1680}P^2 Q\eta^7 + \frac{1}{240}\Pr Q^2\eta^6 + \frac{1}{40}\Pr Q\eta^5 \quad (26)$$

When $m = 2$ equations (17) and (18) becomes

$$f_3(\eta) = -3L_1^{-1}A_2 + 2L_1^{-1}B_2 - L_1^{-1}C_2 \quad (27)$$

$$\theta_3(\eta) = -3L_2^{-1}\Pr D_2 \quad (28)$$

Transforming the polynomials

$$A_2 = f_2\frac{d^2 f_0}{d\eta^2} + f_1\frac{d^2 f_1}{d\eta^2} + f_0\frac{d^2 f_2}{d\eta^2}$$

$$A_2 = -\frac{107}{13440}\eta^8 P^4 + \frac{11}{840}P^2 Q\eta^7 \Pr + \frac{3}{280}P^2 Q\eta^7 - \frac{13}{360}P^2\eta^6 + \frac{1}{48}\eta^6 Q^2 + \frac{1}{8}\eta^5 Q + \frac{1}{6}\eta^4$$

$$B_2 = 2\frac{df_2}{d\eta}\frac{df_0}{d\eta} + \left(\frac{df_1}{d\eta}\right)^2$$

$$B_2 = -\frac{23}{6720}\eta^8 P^4 + \frac{1}{120}P^2 Q\eta^7 \Pr - \frac{1}{24}P^2\eta^6 + \frac{1}{36}\eta^6 Q^2 + \frac{1}{6}\eta^5 Q + \frac{1}{4}\eta^4$$

$$C_2 = \frac{1}{56}P^2 Q\eta^7 \Pr^2 - \frac{1}{1680}P^2 Q\eta^7 \Pr + \frac{1}{240}\Pr Q^2\eta^6 + \frac{1}{40}\Pr Q\eta^5$$

$$D_2 = \frac{1}{16}P^3\eta^8 Q\Pr^2 - \frac{1}{160}P^3\eta^8 Q\Pr - \frac{13}{40320}P^3\eta^3 Q + \frac{19}{560}P\Pr Q^2\eta^7 + \frac{7}{48}PQ\eta^6\Pr + \frac{1}{1008}PQ^2\eta^7$$

Substituting the above polynomials into (27) and (28) and introduce inverse operator

$$f_3(\eta) = \int_0^\eta\int_0^\eta\int_0^\eta (-3A_2 + 2B_2 - C_2)\, d\eta\, d\eta\, d\eta$$

$$\theta_3(\eta) = \int_0^\eta\int_0^\eta -3\Pr D_2\, d\eta\, d\eta$$

Then substituting all the polynomials above to yield

$$f_3(\eta) = \frac{229}{13305600}P^4\eta^{11} - \frac{1}{40320}P^2 Q\eta^{10}\Pr^2 - \frac{37}{1209600}P^2 Q\eta^{10}\Pr + \frac{1}{20160}P^2\eta^9 - \frac{1}{22400}P^2 Q\eta^{10} - \frac{1}{120960}\Pr Q^2\eta^9 - \frac{1}{13440}Q\Pr\eta^8 - \frac{1}{8064}Q\eta^8 - \frac{1}{72576}Q^2\eta^9$$

$$\theta_3(\eta) = -\frac{1}{480}P^3 Q\eta^{10}\Pr^3 + \frac{1}{4800}P^3 Q\eta^{10}\Pr^2 + \frac{13}{1209600}P^3 Q\eta^{10}\Pr - \frac{19}{13440}P\Pr^2 Q^2\eta^9 - \frac{1}{128}\Pr^2 PQ\eta^8 - \frac{1}{24192}P\Pr Q^2\eta^9$$

Using (10), for $m = 0,1,2$ and 3

$$f(\eta) = \frac{1}{2}\eta^2 P + \frac{1}{120}\eta^5 P^2 - \frac{1}{6}\eta^3 - \frac{1}{24}\eta^4 Q - \frac{13}{40320}\eta^8 P^3 + \frac{1}{1008}\eta^7 PQ + \frac{1}{1680}\Pr\eta^7 PQ + \frac{229}{13305600}\eta^{11}P^4 - \frac{1}{40320}\eta^{10}P^2 Q\Pr^2 - \frac{37}{1209600}P^2 Q\eta^{10}\Pr + \frac{1}{20160}\eta^9 P^2 - \frac{1}{22400}\eta^{10}P^2 Q - \frac{1}{120960}\Pr Q^2\eta^9 - \frac{1}{13440}Q\Pr\eta^8 + \frac{1}{8064}Q\eta^8 + \frac{1}{72576}\eta^9 Q^2 \quad (29)$$

$$\theta(\eta) = 1 + \eta Q - \frac{1}{8}\Pr PQ\eta^4 + \frac{1}{56}P^2 Q\eta^7 - \frac{1}{1680}P^2 Q\eta^7\Pr + \frac{1}{240}\Pr Q^2\eta^6 + \frac{1}{40}\Pr Q\eta^5 - \frac{1}{480}P^3 Q\eta^{10}\Pr^3 + \frac{1}{4800}P^3 Q\eta^{10}\Pr^2 + \frac{13}{1209600}P^3 Q\eta^{10}\Pr - \frac{19}{13440}P\Pr^2 Q^2\eta 9 - \frac{1}{128}PQ\eta^8\Pr^2 - \frac{1}{24192}P\Pr Q^2\eta^9 \quad (30)$$

The two assumed values $f''(0) = P$ and $\theta'(0) = Q$ is calculated using bvp4c on MATLAB with the following code

```
function dfdeta = Code1(eta,f,Pr)
Pr=0.72;
dfdeta = [f(2)
          f(3)
          -3*f(1)*f(3)+2*f(2)*f(2)-f(4)
          f(5)
          -3*Pr*f(1)*f(5)];
end
function res = Code2(f0, finf)
res = [f0(1)
       f0(2)
       finf(2)
       f0(4)-1
       finf(4)];
end
>> solinit = bvpinit(linspace(0,10,100),[0
0 0 1 0]);
>> sol = bvp4c(@Code1,@Code2,solinit);
>> sol.y
```

This produced the result as $f''(0) = P = 0.6760$ and $\theta'(0) = Q = -0.5046$. Substituting P, Q and $\Pr = 0.72$ into (29) and (30)

$$f(\eta) = 0.338\eta^2 + 0.003808133333\eta^5 - 0.1666\eta^3$$
$$0.021025\eta^4 - 0.00000999427301\eta^8$$
$$-0.0004845922096\eta^7 + 0.000018337402336\eta^{10}$$
$$+0.000003594080522\eta^{11}$$
$$+0.000017643519896\eta^9 \quad (31)$$

$$\theta(\eta) = 1 - 0.5046\eta + 0.03069986400\eta^4$$
$$-0.002035781076\eta^7 + 0.0007638634800\eta^6$$
$$-0.0090828\eta^5 + 0.0001031703060\eta^{10}$$
$$0.0001312649678\eta^9 + 0.001381493880\eta^8 \quad (32)$$

Figure 1 and 2 shows graphical solution of $f'(\eta)$ against η and $\theta(\eta)$ against η

1.2 Also consider the following coupled system of two ordinary differential equations of first order which shall be solve using Adomian decomposition method

$$\frac{dx}{dt} = t^2 Sin(x) + e^t Cos(y) \quad (31)$$

$$\frac{dy}{dt} = 2tx + e^y \quad (32)$$

Subject to initial conditions

$$x(0) = 1 \text{ and } y(0) = -1 \quad (33)$$

Here $L_1 = L_2 = \frac{d}{dt}$ likewise $L_1^{-1} = L_2^{-1} = \int_0^t (*)dt$ Implies that $L_1 x = \frac{dx}{dt}$ and $L_2 y = \frac{dy}{dt}$.

Applying inverse operator on both sides of (31) and (32)

$$L_1^{-1}(L_1 x) = L_1^{-1}(t^2 Sin(x)) + L_1^{-1}(e^t Cos(y)) \quad (34)$$

$$L_2^{-1}(L_2 y) = L_2^{-1}(2tx) + L_2^{-1}(e^y) \quad (35)$$

The Adomian decomposition method assumes that the unknown function $x(t)$ and $y(t)$ can be expressed by an infinite series of the form

$$x(t) = \sum_{m=0}^{\infty} x_m = x_0 + x_1 + x_2 + \dots + x_\infty$$

$$y(t) = \sum_{m=0}^{\infty} y_m = y_0 + y_1 + y_2 + \dots + y_\infty \quad (36)$$

Linear and nonlinear terms can be decomposed by an infinite series of polynomials given by

$$\sum_{m=0}^{\infty} A_m = \sin(x) \qquad \sum_{m=0}^{\infty} B_m = \cos(y)$$

$$\sum_{m=0}^{\infty} C_m = x_0 \qquad \sum_{m=0}^{\infty} D_m = e^y \quad (37)$$

Where the components, A_m, B_m, C_m, D_m x_m and y_m will be determined recurrently. Also, A_m, B_m, C_m, D_m are the Adomian Polynomials which make a rapidly convergent series. The Adomian polynomial is generated by the following formula (12).The Exact solution is

$$x(t) = \lim_{n\to\infty} \sum_{m=0}^{n} x_m \text{ and } y(t) = \lim_{n\to\infty} \sum_{m=0}^{n} y_m$$

$$L_1^{-1}(L_1 x) = x(t) - x(0),\ L_1^{-1}(L_1 y) = y(t) - y(0) \quad (38)$$

Substitute (33) into (38)

$$L_1^{-1}(L_1 x) = x(t) - 1,\ L_1^{-1}(L_1 y) = y(t) + 1 \quad (39)$$

Substituting (39) into (34) and (35)

$$x(t) = 1 + L_1^{-1}(t^2 Sin(x)) + L_1^{-1}(e^t Cos(y)) \quad (40)$$

$$y(t) = -1 + L_2^{-1}(2tx) + L_2^{-1}(e^y) \quad (41)$$

Equation (40) and (41) can be separated into

$$x_0 = 1 \text{ and } y_0 = -1$$

And

$$x_{m+1} = L_1^{-1}(t^2 Sin(x)) + L_1^{-1}(e^t Cos(y))$$
$$y_{m+1} = L_2^{-1}(2tx) + L_2^{-1}(e^y)$$

Next step is to substitute all the Nonlinear and linear terms

$$x_{m+1} = L_1^{-1}(t^2 \sum_{m=0}^{\infty} A_m) + L_1^{-1}(e^t \sum_{m=0}^{\infty} B_m) \quad (42)$$

$$y_{m+1} = L_2^{-1}(2t \sum_{m=0}^{\infty} C_m) + L_2^{-1}(\sum_{m=0}^{\infty} D_m) \quad (43)$$

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Using the Adomian polynomial formula (12) for $m=0,1$ and 2 on the series (42) and (43)

Starting with when $m=0$

$$x_1 = L_1^{-1}(t^2A_0) + L_1^{-1}(e^tB_0) \quad (44)$$

$$y_1 = L_2^{-1}(2tC_0) + L_2^{-1}(D_0) \quad (45)$$

Using (12) to generate the Adomian polynomials

$$A_0 = \sin(x_0) = \sin(1) \qquad B_0 = \cos(y_0) = \cos(-1)$$

$$C_0 = x_0 = 1 \qquad D_0 = e^{y_0} = e^{-1}$$

Substituting back into equations (44) and (45). Introduce inverse operator

$$x_1 = \int_0^t \left(t^2A_0 + e^tB_0\right)dt \text{ and } y_1 = \int_0^t \left(2tC_0 + D_0\right)dt$$

$$x_1 = \frac{1}{3}t^3\sin(1) - \cos(1) + e^t\cos(1)$$

$$y_1 = t^2 + \cos(1)e^{-1}$$

When $m=1$ equations (42) and (43) becomes

$$x_2 = L_1^{-1}(t^2A_1) + L_1^{-1}(e^tB_1) \quad (46)$$

$$y_2 = L_2^{-1}(2tC_1) + L_2^{-1}(D_1) \quad (47)$$

Using (12) to generate the Adomian polynomials

$$A_1 = x_1\cos(x_0)$$

$$A_1 = \frac{1}{3}t^3\cos(1)\sin(1) - \cos(1) + e^t\cos(1)\cos(1)$$

$$B_1 = -y_1\sin(y_0)$$

$$B_1 = -\sin(-1)t^2 - \sin(-1)\cos(1)e^{-1}$$

$$C_1 = x_1, \qquad D_1 = y_1e^{y_0}$$

$$C_1 = \frac{1}{3}t^3\sin(1) - \cos(1) + e^t\cos(1)$$

$$D_1 = t^2e^{-1} + \left(e^{-1}\right)^2 t$$

Substituting the above polynomials into (46) and (47) and introduce inverse operator

$$x_2 = \int_0^t \left(t^2A_1 + e^tB_1\right)dt \text{ and } y_2 = \int_0^t \left(2tC_1 + D_1\right)dt$$

Then substituting all the polynomials above to yield

$$x_2 = -2\cos(1)^2 + \frac{1}{18}t^2\sin(1)\cos(1) + \cos(1)^2t^2e^t$$
$$-2\cos(1)^2te^t + 2\cos(1)^2e^t - \frac{1}{3}t^3\cos(1)^2$$
$$+e^{-1}\sin(1) - 2\sin(1) - \sin(1)e^{t-1} + 2\sin(1)e^t$$
$$+\sin(1)\sin(1)e^{t-1}t - 2\sin(1)te^t + \sin(1)e^tt^2$$

$$y_2 = 2\cos(1) + \frac{2}{15}\sin(1)t^5 - t^2\cos(1) + 2\cos(1)te^t$$
$$-2e^t\cos(1) + \frac{1}{3}e^{-1}t^3 + \frac{1}{2}e^{-1}t^2$$

For $m = 01,$ and 2 in equation (36)

$$x(t) = 1 + \frac{1}{3}\sin(1)t^3 - \cos(1) + e^t\cos(1) - 2(\cos(1))^2$$
$$+\frac{1}{18}t^6\sin(1)\cos(1) + (\cos(1))^2t^2e^t - 2(\cos(1))^2t^2e^t$$
$$+2(\cos(1))^2e^t - \frac{1}{3}t^3(\cos(1))^2 + e^{-1}\sin(1)$$
$$-2\sin(1) - \sin(1)e^{t-1} + 2\sin(1)e^t$$
$$+\sin(1)e^{t-1}t - 2\sin(1)e^tt + \sin(1)e^tt^2$$

$$y(t) = -1 + t^2 + te^{-1} + 2\cos(1) + \frac{2}{15}t^2\sin(1)$$
$$-t^2\cos(1) + 2\cos(1)te^t - 2e^t\cos(1)$$
$$+\frac{1}{3}e^{-1}t^3 + \frac{1}{2}\left(e^{-1}\right)^2t^2$$

Table 1: This table shows Comparative analysis of f ′(eta) between ADM and Numerical Method

eta	f′(eta) Numerical	f′(eta) AD Method	error f′(eta) |ADM-NUM|
0	0	0	0
0.2	0.1159	0.1159	0
0.4	0.1963	0.1963	0
0.6	0.2461	0.2461	0
0.8	0.2708	0.2708	0
1	0.2759	0.2761	0.0002
1.2	0.2668	0.2675	0.0007
1.4	0.2481	0.2512	0.0031
1.6	0.2237	0.2356	0.0119

Figure 1: This figure shows Comparative analysis of f ′(eta) between ADM and Numerical Method

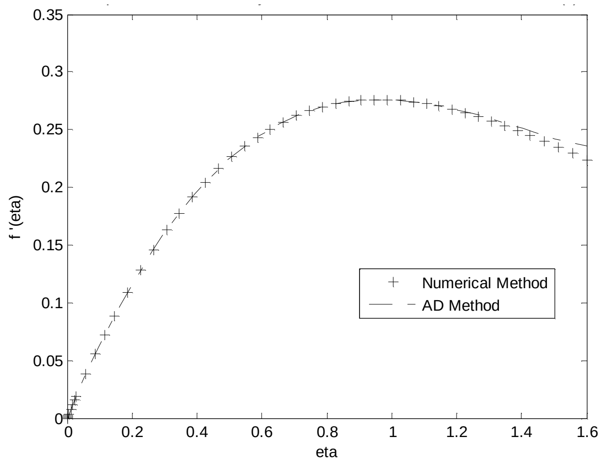


Table 2: This table shows Comparative analysis of theta (eta) between ADM and Numerical Method

eta	θ(eta) Numerical	θ(eta) AD method	error θ(eta) |ADM-NUM|
0	1	1	0
0.2	0.8991	0.8991	0
0.4	0.7989	0.7989	0
0.6	0.7005	0.7005	0
0.8	0.6059	0.6059	0
1	0.5168	0.5171	0.0003
1.2	0.435	0.4364	0.0014
1.4	0.3615	0.3676	0.0061
1.6	0.297	0.3184	0.0214

Figure 2: This figure shows Comparative analysis of theta(eta) between ADM and Numerical Method

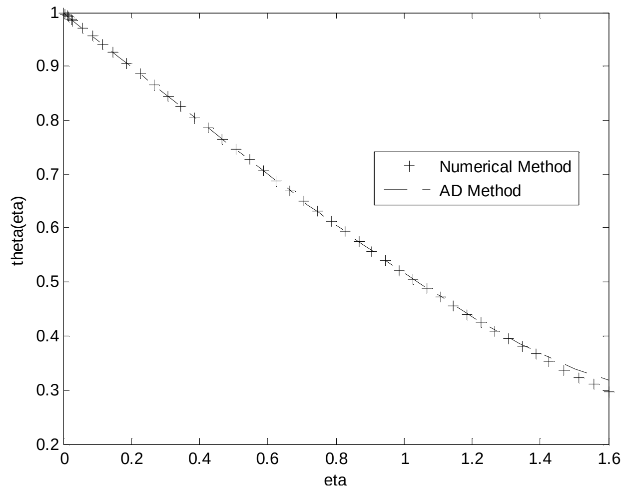


Table 3: This table shows Comparative analysis of x(t) between ADM and Numerical Method

t	x(t)-Numerical Method	x(t)-AD Method	error x(t) |ADM-NUM|
0	1	1	0
0.1	1.0591	1.059069557	3.0443E-05
0.2	1.1318	1.131687662	0.000112338
0.3	1.2244	1.223845854	0.000554146
0.4	1.3438	1.342769849	0.001030151
0.5	1.4977	1.497194555	0.000505445
0.6	1.6915	1.697695741	0.006195741
0.7	1.9231	1.957084244	0.033984244
0.8	2.1707	2.290869491	0.120169491
0.9	2.3727	2.717800055	0.345100055

Figure 3: This figure shows Comparative analysis of x(t) between ADM and Numerical Method

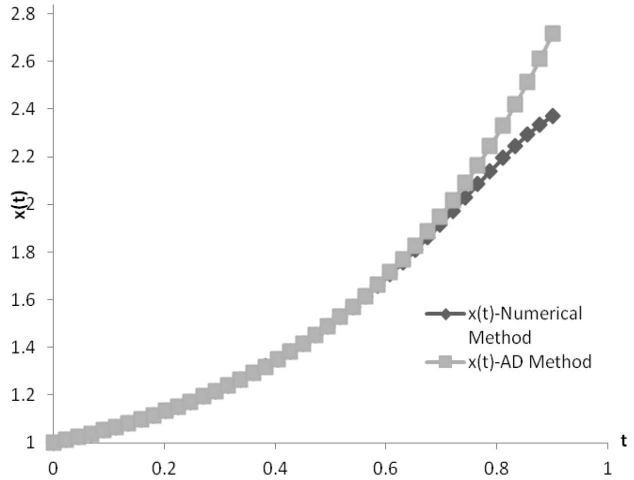
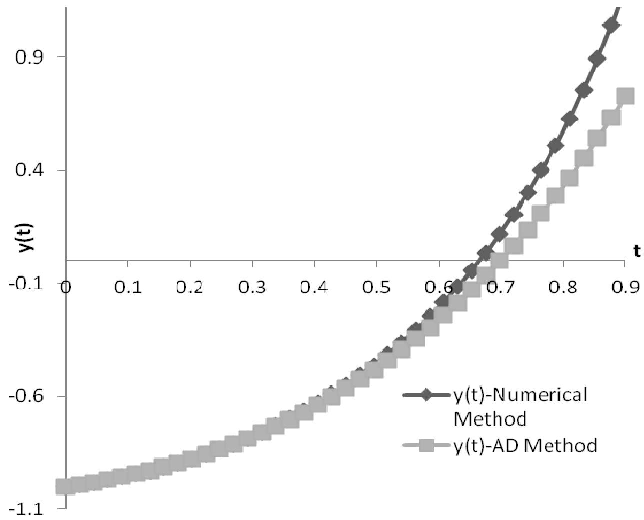


Table 4: This table shows Comparative analysis of y(t) between ADM and Numerical Method

t	y(t)-Numerical Method	y(t)-AD Method	error y(t) |ADM-NUM|
0	-1	-1	0
0.1	-0.952	-0.952037554	3.7554E-05
0.2	-0.8791	-0.879590734	0.000490734
0.3	-0.7765	-0.779049681	0.002549681
0.4	-0.6375	-0.646111764	0.008611764
0.5	-0.4518	-0.475587799	0.023787799
0.6	-0.2047	-0.261200503	0.056500503
0.7	0.1261	0.004625783	0.121474217
0.8	0.5752	0.330985207	-0.244214793
0.9	1.206	0.728721582	-0.477278418

Figure 4: This figure shows Comparative analysis of y(t) between ADM and Numerical Method



CONCLUSION

From the above solutions, Adomian Decomposition has been successfully adopted to solve strong nonlinear boundary differential equations. It's obvious that Adomian Decomposition method can handle such kind of highly nonlinear differential equation. Solutions of ADM are compared with numerical method which excellent convergent (See Fig 1, 2, 3 and 4). From the solutions above, it should be noted that increase in the value of independent value is resulting to a gradual increase in error despite the accelerated in the convergence of ADM solution. As discussed earlier ADM have an easy computational and highly convergent system to solve either linear or nonlinear problems of the real world without assumptions on changing the essential nonlinear nature. The main advantage of Adomian decomposition method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution.

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CONFLICT OF INTEREST

No conflict of interests was declared by authors.