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World J of Engineering and Pure and Applied Sci. 2013;3(1):26 ISSN 2249-0582
Chikwendu and Emenonye, 2013. Dynamic Programming for Optimizing Stock Allocation

Original Article
Applied Science

Dynamic Programming Method Approach for Optimizing Stock Allocation via the Step Function

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ABSTRACT [ENGLISH/ANGLAIS]

Stock control is an important functional branch of any manufacturing/service organization. Stock control has been discussed using the dynamic programming model approach. The distribution of goods to warehouses is a multistage process which the dynamic programming is adapted to providing solution to and also guarantees optimal feasible solution. The stock in warehouses is used as a partition of the firm's main store. An equation ensues from the stock records which is shown to be piecewise continuous. The Laplace transform is used to test the existence of the function. The step function that derives from the supply to the warehouses is transformed to a piecewise linear function which can be approximated as continuous function. The step function is then applied on the function to obtain the returns from the respective allocations if the function exists. The allocation with the highest return is the optimum. Some relevant theorems have been stated and proved and illustrative examples included.

Keywords: Dynamic programming, stock allocation, optimization

RÉSUMÉ [FRANÇAIS/FRENCH]

Le contrôle des stocks est une importante branche fonctionnelle pour toute société industrielle ou de services. Ce système a été examiné en utilisant l'approche dynamique du modèle de programmation. La distribution des marchandises dans les entrepôts est un processus à plusieurs étapes pour laquelle la programmation dynamique est conçue pour apporter une solution et aussi une garantie optimale de solution possible. Le stock dans les entrepôts est utilisé comme une part du magasin principal de l'entreprise. une équation découlant des fiches de stocks a été présentée être un système continue fractionné. La transformation de Laplace est utilisée pour mettre en évidence cette fonction. La fonction par étape qui dérive de l'alimentation des entrepôts est transformée en une fonction linéaire fractionnée qui peut se rapprocher d'une fonction continue. La fonction par étape est ensuite appliquée sur la fonction pour obtenir les rendements des allocations respectives si la fonction existe. L'allocation au fort taux de rendement est la condition optimale. Certains pertinents théorèmes ont été déclarés et prouvés avec inclut des exemples illustratifs.

Mots-clés: Programmation dynamique, allocation de réserve, optimisation

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Accepted/Accepté: November, 2013

Citation: Chikwendu CR, Emenonye C. Dynamic Programming Method Approach for Optimizing Stock Allocation via the Step Function. World Journal of Engineering and Pure and Applied Sciences 2013;3(1)26-30.

INTRODUCTION

Faulty stock allocation has led to huge losses to manufacturing and wholesale firms in the past. In recent times, companies stepped up deliberate effort to minimize the cost incurred from non-availability of their goods in some locations. The conscious effort to ensure that goods reach the consumers has led to warehouses being built/spread at many locations in a region. It is obvious that these activities attract some costs that may affect the amount of profit being made. The stock allocation model seeks to find a stock level that would be maintained in order to optimize returns [1].

Mathematics is a viable tool in attempting to solve this problem of stock allocation. Mathematics model which is a collection of logical and mathematical relationship that represents aspect of the situation is used. In this work a technique for the planning of multistage process, dynamic programming models is used. This model describes a process in term of states, decision, transaction and returns. The process begins in some initial state where a decision is made and this decision causes a transition to a new state. The problem is to find the sequence that maximizes the total return [2].

Goods are stockpiled in different warehouses. The originating warehouse is the main outlet from which the items are sent and stocked in smaller/subsidiary outlet. The subsidiary outlets serve as the sub-rectangles to the main store.

A rectangle is defined as the Cartesian product of two closed intervals [a, b] and [c, d], $Q = [a,b] \times [c,d] = \{(x,y): x \in [a,b] \text{ and } y \in [c,d]\}$ where φ is a rectangle. A function f defined on a rectangle φ is called step function if a partition P and Q exists such that f is constant on each of the open subrectangles of P. A partition P of $[a,b]$ is a finite set of real numbers $(x_0\, x_1, x_2 \ldots x_1)$, such that $a = x_0 \le x_1 \le x_2 \le x_3 \le \cdots \le x_1 = b$ and such that the subintervals therein in P are the intervals $[x_{i-1}, x_1], i = 1, 2, 3, \ldots n.$ The partition of $[a,b]$ is a finite collection of non-overlapping intervals whose union is $[a,b]$.

In this work, the warehouses are the subrectangles and they have regular replenishment and thus supply is constant. The quantity to each warehouse comes in batches which represents the partitions. The step functions that derive from the supply to the warehouses are transformed to piecewise linear function which can be approximated as continuous function whose domain is a closed interval [a, b]. The Laplace transform is used to check for the existence of the resulting functions. If it exists then the function is solved to obtain an optimal allocation.

DEFINITIONS

Stock

Stock is the supply of goods that is available for sale in a warehouse [3].

Dynamic Programming Model Method:

Dynamic programming model is a method of linear optimization that determines the optimum solution of a multivariable problem by decomposing it into stages, each stage comprising a single variable sub problem. It is a recursive equation that links the different stages of the problem in a manner that guarantees that the optimal feasible solution of each stage is also optimal and feasible for the entire problem [4]. The dynamic programming problem to be solved is stated as;

$$\max Z = f_1(x_1) + f_2(x_2) + \cdots + f_n(x_1)$$

Subject to $x_1 + x_2 + \cdots . + x_n \le k$

where k is the number of products allocated and $x_1, x_2 \ldots, x_n \ge 0$ are integral.

Step Function:

Suppose f is a function with Doman in R^p and range in R^q, f is a step function if it assumes only a finite number of distinct values in R^q each non-zero value being taken on an interval in R^P, then f is a step function. f is defined on a rectangle Q of a partition p on Q exists such that f is constant on each of the open sub rectangles of p [5].

Piecewise Linear Function

Suppose g is a function defined on a compact interval [a, b] of R g is called piecewise linear if there are a finite number of points C_r with $a = C_0 < C_1 < C_2 < \cdots . < C_n = b$ and corresponding real numbers $A_r, B_r, \; r = 0, 1, 2, \ldots, n$ such that x satisfies the relation $C_{r-1} < x < C_r$, the function g has the form $g(x) = A_r + B_r, r = 0, 1, 2, \ldots . n.$ Continuous functions can be approximated uniformly by simple functions which are also continuous. Step function by nature are not continuous, hence we adopt a simple approach where p = q = 1 to make them perceives linear functions [6].

Suppose f_p is a given function with domain D contained in R^P and range in R^q. then a function g approximates f uniformly on D within $\varepsilon > 0$ if

$$\| g(x) - f(x) \| \le \varepsilon \qquad \forall x \text{ in } D.$$

Laplace Transform

Let f be a function defined for $t \ge o$. then the integral $\mathcal{L}(f(x)) = \int_o^\infty e^{-st} f(x)\,dt$ is said to be the Laplace transform of f provided the integral converges. When $\mathcal{L}(f(x)$ converges, the result is a function of S. The Laplace transform is a method where operation of calculates on function are replaced by operation of algebra on transforms. The Laplace transform has a linearity property which makes it very useful for solving linear differential equation with constant coefficients.

THEOREMS

1. Let f be a continuous function whose domain a compact cell is in R^P and whose values belong to R^q. Then f can be uniformly approximated on D by step function [7].

Proof

Assume $\varepsilon > 0$ be given

But it is true that if f is a continuous function with domain D in R^p and range in R^q and if $N \subseteq D$ is compact, then f is uniformly continuous on N.

$\therefore$ there is a number $\delta(\varepsilon) > 0$ such that if $x < y$ belong to D and $\|x - y\| < \delta(\varepsilon)$, then $\|f(x) - f(y)\| < \varepsilon$.

Now divide the domain D of into disjoint cells $I_1, I_2, \ldots, I_n$ such that x, y belong to $I_k, k = 1, 2, \ldots n$ and define $g(x) = f(x_k)$ for $x \text{ in } I_k$ and $g(x) = 0$ for x not in D.

i.e. $$g(x) = \begin{cases} x_k \text{ if } x \in D \\ 0 \text{ if } x \notin D \end{cases}$$

Then clearly $\|g(x) - f(x_k)\| < \varepsilon$ for x in D.

So that g approximates f uniformly on D within ε.

2. Suppose f is a continuous function whose domain is a closed interval $[a, b]$, then f can be uniformly approximately on [a, b] by continuous piecewise linear function.

Proof

The uniform continuity of f is compact on the cell [a, b].

$\therefore$ Given $\varepsilon > 0$, we can partition [a, b] into cells by adding intermediate points $C_r, r = 0, 1, 2, \ldots, n$ with $a = C_0 < C_1 < C_2 < \cdots < C_n = b$ so that $C_r - C_{r-1} < \delta(\varepsilon)$.

If the points $(C_r, f(C_r))$ are connected by line segments and define the resulting continuous piecewise linear function g.

Thus clearly, g approximates f uniformly on [a, b] with ε

3. Laplace transform is a linear operation i.e. for any function $f(x)$ and $g(x)$ whose transform exist and any constants a and b, the transform of $af(x) + bg(x)$ exist and
$$\mathcal{L}(af(x) + bg(x)) = a\,\mathcal{L}[f(x)] + b[g(x)]$$ [8]

Proof

By definition $F[s] = \mathcal{L}(t) = \int_0^\infty e^{-st} f(t)dt$ (1)

$\therefore \mathcal{L}[af(t)] = \int_0^\infty ae^{-st} f(x)dt, \quad \mathcal{L}[bg(t)] = \int_0^\infty be^{-st} g(t)dt$ (2)

$$\Rightarrow \mathcal{L}[af(t) + bg(x)] = \int_0^\infty ae^{-st} f(t)dt + \int_0^\infty be^{-st} g(t)dt$$

$= a\int_0^\infty e^{-st} f(t)dt + b\int_0^\infty e^{-st} g(t)dt$ (3)

From (1) above, (3) becomes

$a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$

$\therefore \mathcal{L}[af(t) + bg(t)] = a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$.

4. If $f(t)$ is piecewise continuous on $[0, \infty)$ and of exponent order c for $t > T$, 1
Then $\mathcal{L}[f(t)]$ exists for $for\ s > c$. [8]

Proof

$\mathcal{L}[f(t)] = \int_0^T e^{-st} f(t)dt + \int_T^\infty e^{-st} f(t)dt = I_1 + I_2$.

The integral I_1 exists because it can be written as a sum of integrals over intervals on which $e^{-st} f(t)$ is continuous.

Now $|I| \le |e^{-st} \int f(t)|dt \le m\int_T^\infty e^{-st} e^{ct} dt$

$= m\int_T^\infty e^{-(t-c)ti} dt$

$$= -m\left[\frac{e^{-(t-c)s}}{s-c}\right]_T^\infty = m\frac{e^{-(s-c)T}}{s-c}$$

For $s > c$

Since $\int_T^\infty me^{-(s-c)t}dt$ converges, the integral $\int_T^\infty |e^{-st} f(t)|dt$ converges by the comparison test for improper integrals.

$\Rightarrow$ T_2 exists for $s > c$

$\therefore$ the existence of I_1 and I_2 implies that

$\mathcal{L}[f(t) = \int_T^\infty e^{-st} f(t)dt$ exists for $s > c$

EXAMPLES

A firm has three warehouses through which it distributes from the depot. The following is the record of batches supplied to the warehouses and their corresponding costs incurred. Obtain the optimum allocation of the company.

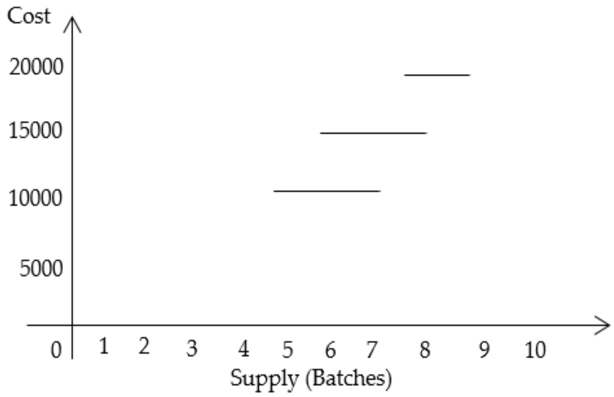
(a) Warehouse Cost (units) supply (batches in 000)

A. 10,000 if 50 $\le t \le 70$

B. 15, 000 if 75 $\leq t \leq 90$

C. 12,000 if 60 $\leq t \leq 80$

The graph appears thus:



The function for the supply is $f(t) = f(x_1) + f(t_2) + f(t_3)$. This function is assumed piecewise continuous hence apply the Laplace transform to test existence. i.e.

$$\mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$\int_{50}^{70} 10{,}000e^{-st}\, dt + \int_{75}^{90} 15{,}000e^{-st} dt + \int_{60}^{80} 12{,}000e^{-st} dt$$

$$= \left[\frac{10{,}000e^{-st}}{-s}\right]_{50}^{70} + \left[\frac{15{,}000e^{-st}}{-s}\right]_{75}^{90} + \left[\frac{12{,}000}{-s}\right]_{60}^{80}$$

$$= \frac{1}{-s}\{[10{,}000e^{-st}]_{50}^{70} + [15{,}000e^{-st}]_{75}^{90} + [12{,}000e^{-st}]_{60}^{80}$$

$$= \frac{1}{s} e^{56/25}$$

Since $s > 0 \quad \lim_{s \to \infty} \frac{1}{-s} e^{2.24}$ exists. We can obtain the respective returns i.e.

$$\int_{50}^{70} 10{,}000\, dx = [10{,}000x]_{50}^{70} = 200{,}000$$

$$\int_{50}^{70} 15{,}000\, dx = [15{,}000x]_{75}^{90} = 225{,}000$$

and

$$\int_{60}^{80} 12{,}000\, dx = [12{,}000x]_{60}^{80} = 240{,}000$$

The returns are 200,000, 225,000 and 240,000 respectively. Warehouse C has the best and hence the optimal allocation is 60,000 to 80,000 batches

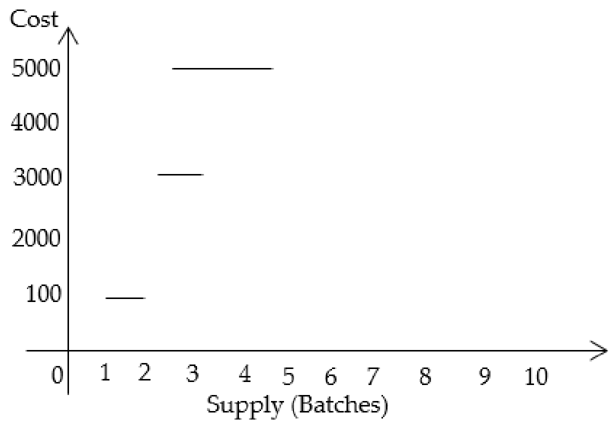
(b) Warehouse Cost (units) supply (batches in 000)

A. 1,000 if 10 $\leq x \leq 20$

B. 3, 000 if 20 $\leq x \leq 30$

C. 5,000 if 30 $\leq x \leq 50$

The graph appears thus:



Using Laplace transform to ascertain existence or otherwise gives the following

$$\int_{10}^{20} 1{,}000e^{-st}\, dt + \int_{20}^{30} 3{,}000e^{-st} dt + \int_{30}^{50} 5{,}000e^{-st} dt$$

$$= \left[\frac{1{,}000e^{-st}}{-s}\right]_{10}^{20} + \left[\frac{3{,}000e^{-st}}{-s}\right]_{20}^{30} + \left[\frac{5{,}000}{-s}\right]_{30}^{50}$$

$$= \frac{1}{s} e^{\frac{20}{10}} + e^{\frac{30}{20}} + e^{\frac{50}{30}}$$

$s > 0, \lim_{s \to \infty} \frac{e^{s}}{-s}$ exist

Hence

$$\int_{10}^{20} 1{,}000\, dx = [20{,}000 - 10{,}000 = 10{,}000$$

$$\int_{20}^{30} 3{,}000\, dx = 90{,}000 - 60{,}000 = 30{,}000$$

$$\int_{30}^{50} 5{,}000\, dx = 250{,}000 - 150{,}000 = 100{,}000$$

The highest return is 100,000 and hence the optimum allocation is 30,000 to 50,000 batches of warehouse Z.

CONCLUSION

Stock control is an important functional branch of any organization and needs proper management [10]. Stock allocation has been x-rayed using the step function as an approximation method. The given stock allocation are represented graphically to show that it is a piecewise function. A step function is not continuous but can be approximated using the piecewise continuous linear functions [9]. When a variety allocation of stock to different warehouses are presented, the Laplace transform is applied to ascertain the existence of the function that results from the allocation to the warehouses. If the function exists, then the unit step function is applied to calculate the corresponding return of each channel. The allocation with the highest cost is the optimal.

Some effort has been made to apply some mathematical tools in stock allocation. Stock allocation has been medelled such that the step function can be used to obtain the approximate optimal allocation.

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ACKNOWLEDGEMENT / SOURCE OF SUPPORT

Nil.

CONFLICT OF INTEREST

No conflict of interests was declared by authors.