Modelling and Simulation of Low Calorific Value Municipal Solid Waste Incineration

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ABSTRACT [ENGLISH/ANGLAIS]

The paper focused on the mathematical modelling and simulation of low calorific value municipal solid waste incineration in semi-arid region. Municipal solid wastes were collected and analysed; from which wood samples were taken and incinerated using a prototype incinerator. A mathematical model, based on Fourier series (FS) and finite elements method (FEM) was developed, ran and validated using experimental and literature data. Critical minimum volume of 4.93 m³ or about 6% of the total volume of the incinerator would be needed at 20% moisture content refuse and a critical maximum volume of 51.10 m³ or 69.21% of the total volume, would be needed at 50% moisture content. The ignition time and the time at which the combustion becomes self-sustaining are greatly influenced by the moisture content. Critical drying temperature was found to be 711.44 K. The grid size had little or no effect on the simulated results. The developed model can accurately predict the critical sizes and performance of the actual plant even at its virtual stage. There is need to optimize the critical drying temperature and mathematical model /physical model correlation which would help improve the efficiency of the process and better understand the drying mechanism involved.

Keywords: Municipal solid waste, calorific value, drying, mathematical modelling, finite element method, Fourier series, simulation

INTRODUCTION

In most cities of many developing countries (such as Kano, in Nigeria, Bandung, in Indonesia, etc.), wastes usually contain 40 to 80% moisture [1]. This results from the tropical climate and the inadequacy of the current waste separation system if at all available. This high moisture content is responsible for the low calorific value; a higher combustion time (i.e.: residence time) in furnaces as well as combustion instability and low efficiency [2]. Drying, is an important and crucial stage in incineration.[3-8]. Drying of food, mainly potatoes at low temperatures (less than 100°C) [6] and most recently mining products [7] were undertaken on two and three dimensional basis respectively at low temperatures. Dincer and Hussain [9] modelled the drying of cylindrical products using the Fickian equation and the Fourier equation of heat transfer. Bellagi et al. [4] modelled and simulated the drying phenomena with rheological behaviour while Porto and Lisboa [7]
modelled the rate of moisture removal in parallelepiped Brazilian oil shale particle at low temperatures (between 60 and 90°C).

This paper treats the development and simulation of a mathematical model of municipal solid waste (MSW) and its validation using a physical model built to that effect as prototype.

MATERIALS AND METHODS

Materials

Wood samples, a small size test tunnel [10], a digital weighting balance PHILLIPS (range 20kg ± 0.001g), a digital stop watch, four (4) digital thermocouples: one (1) DRETEC model (ranges of -50 -300°C); one (1) RKC model REX- C 700FK02-R*KN range 0-400°C; one (1) YUYAO thermocouple model TCA- 6372P ranges 0-500°C and one (1) NAAR-WRB of temperatures range (0-1600°C ± 1.5°C or ± 0.25%). A HITACHI air compressor (Model: EFOU 1 range 0-20 kgfcm⁻²) set at 3.9 kgfcm⁻² say 4 bar and a HP Pavilion Laptop dv6700 Intel (R) Core (TM) 2 Duo CPU T5750 @ 2.00 GHz, 2.00 GB (RAM) and 160 GB (HD).

Methods

Wood samples labelled A, B, C and D (Table 1) were immersed into a single water bath to induce moisture at each sample’s moisture absorption potential. The samples were weighted daily to check the amount of moisture induced. They were later completely dried in an electric oven set at 103 ± 2 °C for 24 hours as mentioned by Walker et al. [11] and then weighted again. The weighting balance was then lagged with a wood insulator to prevent it from getting heated up while placed on top of the drying tunnel. The sample was then hung onto the weighting balance by a thin wire of known weight. Each sample in every category was placed in the test tunnel and tested for drying. A Propane-Butane gas flame was used for the drying since it gives about 1977°C (i.e.: 2250 K) [12, 13].

The percentage reduction in weight was recorded along with the temperature increase at various positions. Testing was stopped as soon as the initial weight before moisture induction was reached. A Matlab M-file package program was used to analyse and generate results for all the grid sizes, by iteration and Fourier transforms.

Table 1: This table shows initial test parameters and samples characteristics

<table>
<thead>
<tr>
<th>Samples</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Carpenter’s plank (soft wood)</td>
<td>Thrown away tree trunk</td>
<td>Discarded fire wood</td>
<td>Dead tree branch</td>
</tr>
<tr>
<td>Size</td>
<td>10 x 10 x 10 cm</td>
<td>Irregular</td>
<td>Irregular</td>
<td>Irregular</td>
</tr>
<tr>
<td>Ambient temperature (K)</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
<tr>
<td>Initial weight (grams)</td>
<td>338</td>
<td>130</td>
<td>258</td>
<td>39</td>
</tr>
<tr>
<td>Weight of wire + hook (grams)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Moist weight (grams)</td>
<td>512</td>
<td>187</td>
<td>341</td>
<td>52</td>
</tr>
<tr>
<td>After drying weight (grams)</td>
<td>335</td>
<td>128.56</td>
<td>256</td>
<td>37.9</td>
</tr>
<tr>
<td>Moisture absorption (%)</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

MATHEMATICAL MODEL FORMULATION

The drying of a waste particle like all other wet porous solid drying involves a simultaneous, coupled heat and mass transfer phenomena [14, 9, 15]. The model of the drying process is based on the Fourier law of heat conduction and the Fick’s law of diffusion [16] equations (1) and (2) respectively:

\[
\left( \frac{q}{A} \right)_x = -k \frac{\partial T}{\partial x} 
\]

\[
\frac{m_c}{A} = -D \frac{\partial C_A}{\partial x}
\]

The Laplacian of the heat equation is expressed as:
\[
\frac{1}{\alpha} \frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}
\]  

(3)

Using the method of separation of variables, the product solution is of the form [14, 16]:

\[
F(x, y, z, t) = F(xyz)G(t)
\]

(4)

Boundary conditions are expressed as equations (5-7).

\[x=0 \Rightarrow T(0, y, z, t) = 0 ; x=L \Rightarrow T(L, y, z, t) = f(x) \]

(5)

\[y=0 \Rightarrow T(x, 0, z, t) = f(x) ; y=Y \Rightarrow T(x, L, z, t) = f(x) \]

(6)

\[z=0 \Rightarrow T(x, y, 0, t) = f(x) ; z=Z \Rightarrow T(x, y, L, t) = f(x) \]

(7)

The dimensionless temperature \((T^*)\) and moisture \((M^*)\) content can be written as [16]:

\[
T^* = \frac{T - T_i}{T_d - T_i}
\]

(9)

\[
M^* = \frac{M - M_d}{M_i - M_d}
\]

(10)

The solution of the three dimensional equation can be obtained as a product of one dimensional problem [14]. Using dimensionless variables, the function \(T^*\) is such that:

\[
T^* (\varepsilon_x, \varepsilon_y, \varepsilon_z, \chi_x, \chi_y, \chi_z) = T^* (\varepsilon_x) T^* (\varepsilon_y) T^* (\varepsilon_z)
\]

(11)

The new function is now:

\[
\frac{\partial T^*}{\partial \xi_i} = \alpha \left[ \frac{\partial^2 T^*}{\partial \xi_i^2} \right]
\]

(12)

The dimensionless variables are given in equations (13-15):

\[\varepsilon_z = \frac{z}{L_z} ; \quad \zeta_z = \frac{\alpha}{L_z^2} t ; \quad T_z^* = \frac{T - T_i}{T_d - T_i}
\]

(13)

\[\varepsilon_y = \frac{y}{L_y} ; \quad \zeta_y = \frac{\alpha}{L_y^2} t ; \quad T_y^* = \frac{T - T_i}{T_d - T_i}
\]

(14)

\[\varepsilon_x = \frac{x}{L_x} ; \quad \zeta_x = \frac{\alpha}{L_x^2} t ; \quad T_x^* = \frac{T - T_i}{T_d - T_i}
\]

(15)

**RESULTS**

To be able to relate the experimental results with the model generated results, there was need to convert the former into the dimensionless form.

**Mathematical Model Solution**

The solutions of the heat and mass transfer problem were obtained by solving the infinite Fourier series (16) [20, 21] and adapting the mass transfer solution of Porto and Lisboa [7].

\[
T^*_n (\varepsilon_z, \zeta_z) = \sum_{n=1}^{\infty} T^*_n (\varepsilon_z, \zeta_z) = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{(2 \lambda_n + \sin 2 \lambda_n)} \cos (\lambda_n \varepsilon_z) e^{-\lambda_n \zeta_z}
\]

(16)

Such that \(\lambda_n [7]:\)

\[
\tan \lambda_n + \frac{\text{Bi}}{\lambda_n} = 0
\]

(17)
Equation (16) constitutes the solution approach to the heat transfer model. Using the experimental and thermo physical properties of the drying material; the diffusion coefficient for tropical woods [22-24] and the dimensionless numbers were obtained (Table 2).

Table 2: This table shows Calculated Dimensionless Numbers of refuse in this study

<table>
<thead>
<tr>
<th>Dimensionless Number</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds Number (Re)</td>
<td>451,520</td>
</tr>
<tr>
<td>Schmidt number (Sc)</td>
<td>6.12 x 10⁶</td>
</tr>
<tr>
<td>Sherwood Number (Sh)</td>
<td>53.739</td>
</tr>
<tr>
<td>Biot Number (Bi)</td>
<td>Heat transfer: 1.61 x 10⁴</td>
</tr>
<tr>
<td></td>
<td>Mass transfer: 1.92 x 10⁻¹²</td>
</tr>
</tbody>
</table>

Mathematical model generated results

The value of λₙ for the dimensionless temperature was found to be 0.8388, while the value of λₙ for the dimensionless moisture was found to be 0.1002. The mathematical drying model enabled the determination of dimensionless moisture (M*) and temperature (T*) as shown in Table 3 and Figure 1.

Table 3: This table shows predicted dimensionless temperature and moisture for various grids.

<table>
<thead>
<tr>
<th>Moisture</th>
<th>11 x 11 x 11</th>
<th>21 x 21 x 21</th>
<th>31 x 31 x 31</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T*</td>
<td>M*</td>
<td>T*</td>
</tr>
<tr>
<td>1.00</td>
<td>2.2773</td>
<td>3.6407</td>
<td>1.00</td>
</tr>
<tr>
<td>0.9100</td>
<td>2.4277</td>
<td>3.6441</td>
<td>0.952</td>
</tr>
<tr>
<td>0.820</td>
<td>2.5881</td>
<td>3.6474</td>
<td>0.9048</td>
</tr>
<tr>
<td>0.730</td>
<td>2.7590</td>
<td>3.6507</td>
<td>0.857</td>
</tr>
<tr>
<td>0.640</td>
<td>2.9412</td>
<td>3.6541</td>
<td>0.8096</td>
</tr>
<tr>
<td>0.550</td>
<td>3.1355</td>
<td>3.6574</td>
<td>0.7620</td>
</tr>
<tr>
<td>0.460</td>
<td>3.3426</td>
<td>3.6607</td>
<td>0.710</td>
</tr>
<tr>
<td>0.370</td>
<td>3.5634</td>
<td>3.6641</td>
<td>0.660</td>
</tr>
<tr>
<td>0.280</td>
<td>3.7988</td>
<td>3.6674</td>
<td>0.610</td>
</tr>
<tr>
<td>0.190</td>
<td>4.0497</td>
<td>3.6708</td>
<td>0.540</td>
</tr>
<tr>
<td>0.10</td>
<td>4.3172</td>
<td>3.6741</td>
<td>0.4924</td>
</tr>
<tr>
<td>0.01</td>
<td>4.6023</td>
<td>3.6775</td>
<td>0.446</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.3972</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.3496</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.253</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0640</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0476</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Figure 1:** This figure shows comparative chart of predicted and experimental dimensionless temperatures.

**Figure 2:** This figure shows weight variation, temperature, time relationship.

**Table 4:** This table shows curve equations table for the mathematical model and physical model results.

<table>
<thead>
<tr>
<th>Fitted curves</th>
<th>$R^2$</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical model</td>
<td>0.983</td>
<td>Eq.(29)</td>
</tr>
<tr>
<td>Sample category A</td>
<td>0.995</td>
<td>Eq.(30)</td>
</tr>
<tr>
<td>Sample category B</td>
<td>0.993</td>
<td>Eq.(31)</td>
</tr>
<tr>
<td>Sample category C</td>
<td>0.996</td>
<td>Eq.(32)</td>
</tr>
<tr>
<td>Sample category D</td>
<td>0.992</td>
<td>Eq.(33)</td>
</tr>
</tbody>
</table>

In order to effect a transition between the actual incinerator [13] and the prototype, use is made of equation (34):

$$ C = \eta_y \times \eta_{yr} $$

The correlation (Equation 34) achieved constitutes the relationship between the physical model and the mathematical model, in order to determine the critical sizes (minimum and maximum) of the incinerator.

**Table 5:** This table shows summary of incinerator critical sizes.

<table>
<thead>
<tr>
<th></th>
<th>Critical length (m)</th>
<th>Critical height (m)</th>
<th>Critical breath (m)</th>
<th>Volume (m$^3$)</th>
<th>Percentage of total volume (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% moisture</td>
<td>4.25</td>
<td>1.00</td>
<td>1.16</td>
<td>4.93</td>
<td>5.99</td>
</tr>
<tr>
<td>50% moisture</td>
<td>9.75</td>
<td>2.21</td>
<td>2.65</td>
<td>57.10</td>
<td>69.21</td>
</tr>
</tbody>
</table>

**DISCUSSION**

The mathematical drying model of the numerical bed type developed was ran and validated with experimental data and data from past researches’ results as well as physical model generated data. Experimental and predicted dimensionless temperatures are showing a convergence as could be observed in Table 4, where the $R^2$ values are very close showing that there is a very good agreement between the physical and mathematical models. Likewise it shows an accurate prediction of the temperature and moisture by the mathematical model.

Also, sample (A) (soft wood) was found to absorb the highest moisture of 50%; this could be due to the fact that the tree was alive and its fibres were still active and growing when the tree was brought down, or the treatment it might have undergone during its processing at the wood factory. The smaller the moisture content, the lesser the time it takes to dry up, which further signifies the influence of the sample’s moisture content on the drying process within an incinerator. At the beginning of the processes (i.e.: physical and mathematical models), the initial or input conditions
were always fixed. The divergence could be as a result of error or a clear picture of the test conditions. In all cases, after the first experiment, the temperature within the tunnel chamber was found to have increased. Cooling of the chamber was then allowed for the temperature to reach a near ambient one, corresponding to the initial temperature before the first experiment was even carried out.

Because of some perceived inadequacy of the available thermocouple, some relatively small degree of errors might have been incorporated. These could also be as a result of an initial weight loss (oven drying weight) observed; due to the burning of some parts of the samples as a result of exposure to direct flame.

With increase in temperature and reduction in moisture, both the physical and mathematical models show a very good convergence as mentioned previously. However, after the first experiment, the temperature patterns changed slightly, possibly because of the walls high temperature and that of the air within the test tunnel. The thermocouples readings were high while the moisture removal rate was low compared to the first experiment conducted. This explains the closeness of the experimental graphs of sample in categories B, C and D to the mathematical model predicted graph. To this should be added the fact that moisture removal for samples in categories B, C and D started almost in the same temperature ranges of 473 K. The mathematical model satisfactorily predicted the upper and lower boundaries of both the temperature and moisture. The fitted curves for the mathematical and physical models were very similar and satisfactory (Figure 1). This can be interpreted to mean a very good agreement between the results of the predicted mathematical and physical models.

The intersection of curves in Figure 2 proved that the critical characteristics of the drying process are: $T_r = 762$ K, $t_r = 517$ sec., $M_r = 90.02$ grams; while the critical pressure of the physical model was then found to be $P_r = 11.90$ bar. This occurs when the drying process is 51.72% complete: meaning that only 51.72% of the moisture is taken away. It does not represent the total drying process because this represents a fraction of the samples used during the experiments, but gives the general behaviour of the system.

The breadth of the actual bed is 3 m, far beyond the minimum critical breath of 1.16 m and the maximum critical breath of 2.65 m; thus the refuse could be spread all along the 3 m, so as to eventually narrow the height to be used. However, it should be noted that for these to take place, the temperature must be greater or equal to 762 K, which can conventionally be easily achieved through the injection of extra start up fuel in the form of gas, or residual fuel oil.

It was indeed shown by the mathematical model that drying was induced far above the temperature of 313 K, around 711.44 K. Calculations show that all the physical and thermal processes are confined to a narrow band of 1.00 m along the bed height. An initial half meter (1/2 meter) along the bed length, needs to be considered since during loading, conditions are near ambient ones, therefore no process may take place. The devolatilisation process would start and the char burning rate would also rise to a peak level as all the oxygen ($O_2$) would be available for combustion. The whole combustion process would then be completed before the discharge point.

This result is in good agreement with Changkook et al. [25] who simulated a 10 ton/h throughput of waste, centre flow Martin grate type MSW incinerator using FLIC and FLUENT software. Even though, the refuse characteristics are: Moisture: Volatile Matter: Fixed Carbon: Ash content in waste as 36: 82: 23: 8 respectively and a lower calorific value of 7.65 MJ/kg, which are different from the present work. They observed that at a location near the 2m position, a very sharp rise in the devolatilisation rate occurs as the bed temperature is raised above the threshold (260 °C) and volatile gases start to release from the solids. Char begins to be formed and starts to burn at a position of 2.3 m along the bed length. At 6.3 m, all the moisture in the solids is evaporated while the whole combustion process is completed at 7.5 m.

Real operating conditions may be slightly different from those obtained here due to refuse components such as rags, paper, and all lighter materials contained. However, these conditions are the optimum ones and conditions may differ since wood, amongst other components, in the area of study usually contains more moisture. The waste moisture content and the size of the particles have a direct influence on the combustion. Combustion time is greatly influenced as it increases with an increase in moisture content, irrespective of the primary or secondary air.

**CONCLUSION**

This study concludes as follows:

1. A three dimensional drying model of the numerical bed type, for the semiarid regions was developed, ran and validated using Matlab 7.7 and compared with data
obtained from both experimental and results from other researchers.
2. Critical minimum volume of 4.93 m$^3$ or about 6% of the total volume of the incinerator would be needed for 20% moisture content refuse and a critical maximum volume of 51.10 m$^3$ or 69.21% of the total volume, would be needed for 50% moisture content (i.e. saturation point of wood).
3. The ignition time and the time at which the combustion becomes self sustaining are greatly influenced by the moisture content. Critical drying temperature was found to be 711.44 K.
4. The grid size had little or no effect on the simulated results. The developed model can accurately predict the critical sizes and performance of the actual plant even at its virtual stage.
5. Optimization of the critical drying temperature (i.e. 711.44 K) and mathematical model/physical model correlation would help improve the efficiency of the process and better understanding of the drying mechanism involved.

NOMENCLATURE

\[ V_p \] = volume of the prototype, m$^3$

\[ V_m \] = volume of the model, m$^3$

\[ W_p \] = moisture in the prototype, kg

\[ W_m \] = moisture in the model, kg

\[ h \] = film coefficient or heat transfer coefficient or convective heat transfer coefficient

\[ L_c \] = characteristic length, which is commonly defined as the volume of the body divided by the surface.

\[ k_b \] = thermal conductivity of the body.

\[ h_m \] = film mass transfer coefficient

\[ L_c \] = characteristic length

\[ D_{ab} \] = mass diffusivity

REFERENCES


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CONFLICT OF INTEREST
Nil

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