ABSTRACT [ENGLISH/ANGLAIS]

Modelling road traffic could be used to determine vital characteristics such as speed and density at which maximum flow occurs and the jam density of a roadway facility. The study focuses on modelling the road traffic data collected on three spots along Zaria-Kaduna highway. The data was collected by photography and stop-watch methods on referenced strips of the highway. Macroscopic approach-Greenshields and Greenberg models were applied and regression analysis was used to compare the models. The results show superior performances of Greenberg model over Greenshields models since the former has higher $R^2$ values over the latter, but the contrast is the inability of Greenberg's model to predict speeds at densities nearing zero. Also, Greenberg's model is a better predictor of jam density than Greenshield's model.

Keywords: Speed, density, flow, model, vehicular traffic

RÉSUMÉ [FRANÇAIS/FRENCH]

Modélisation du trafic routier pourrait être utilisée pour déterminer les caractéristiques essentielles telles que la vitesse et la densité à laquelle se produit un débit maximal et la densité de la confiture d'une installation de la chaussée. L'étude se concentre sur la modélisation des données de circulation routière collectées sur trois points le long de Zaria-Kaduna autoroute. Les données ont été recueillies par des méthodes de photographie et d'arrêt-montre sur les bandes référencées de la route. Modèles macroscopiques approche-Greenshields et Greenberg ont été appliquées et l'analyse de régression a été utilisée pour comparer les modèles. Les résultats montrent des performances supérieures du modèle de Greenberg sur les modèles Greenshields puisque le premier a des valeurs plus élevées que la $R^2$-ci, mais le contraste est l'incapacité du modèle de Greenberg pour prédire les vitesses à des densités approchant de zéro. En outre, le modèle de Greenberg est un meilleur prédicteur de la densité de la confiture que le modèle de Greenshield.

Mots-clés: Vitesse, la densité, le débit, le modèle, la circulation des véhicules

INTRODUCTION

Vehicular traffic volume has rapidly outstripped the capacities of the nation’s highways and the problem of limited road capacity has been growing. It has become increasingly necessary to understand the dynamics of traffic flow and obtain mathematical description of the processes. The fundamental relationship between flow and density was first proposed by Greenshields [1]. Traffic phenomena have been modeled as flow from the analogy between vehicular traffic and flow which is continuum [2-5].

Macroscopic modeling looks at traffic flow from a global perspective, whereas microscopic modeling gives attention to the details of traffic flow and the interactions taking place within it [6]. The modeling approaches have provided useful tools to understand traffic phenomena such as the formation and dissipation of traffic jams [7].

The major advantage of macroscopic models is their tractable mathematical structure and their low number of parameters [8].

MATERIALS AND METHODS

Macroscopic modeling is based on the idea of Lighthill and Whitham [9] equation of fluid dynamics describing the flow of fluids could also describe the motion of cars along a road, provided a large-scale point of view is adopted so as to consider cars as small particles and their density as the main quantity to be looked at. This analogy remains nowadays in all macroscopic models of vehicular traffic, as terms like traffic pressure, traffic flow, traffic waves demonstrate.

The primary elements of traffic flow are flow, density, and speed. Another element, associated with density, is...
the gap or headway between two vehicles in a traffic stream.

The definitions of these elements according to [10] are as follows:

i. \( \text{Flow} (q) \) is the equivalent hourly rate at which vehicles pass a point on a highway during a time period less than 1 hour. It can be determined by:

\[
q = \frac{n \times 3600}{T} \quad (\text{Veh/h})
\]

where \( n \) = the number of vehicles passing a point in the roadway; \( T \) = the equivalent hourly flow and \( T \) = Time in secs.

ii. \( \text{Density} (k) \), sometimes referred to as concentration, is the number of vehicles travelling over a unit length of highway at an instant in time. The unit length is usually 1 mile (mi) thereby making vehicles per mile (veh/mi) the unit of density.

iii. \( \text{Speed} (u) \) is the distance travelled by a vehicle during a unit of time. It can be expressed in miles per hour (mi/h), kilometre per hour (km/h), or feet per second (ft/sec).

iv. \( \text{Time mean speed} (\bar{u}) \) is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time. The time mean speed is found by:

\[
\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i
\]

where \( n \) = number of vehicles passing a point on the highway; \( u_i \) = speed of the \( i \)th vehicle in (ft/sec)

v. \( \text{Space mean speed} (\bar{u}_s) \) is the harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time. It is obtained by dividing the total distance travelled by two or more vehicles on a section of highway by the total time required by these vehicles to travel that distance. This is the speed that is involved in flow-density relationships. The space mean speed is found by:

\[
\bar{u}_s = \frac{n}{\sum_{i=1}^{n} \frac{1}{u_i}} = \frac{uL}{\sum_{i=1}^{n} \frac{L}{u_i}}
\]

where space mean speed (ft/sec); \( n \) = number of vehicles; \( t_i \) = the time it takes the \( i \)th vehicle to travel across a section of highway (sec); \( u_i \) = speed of the \( i \)th vehicle in (ft/sec); \( L \) = length of section of highway (ft).

vi. \( \text{Time headway} (h) \) is the difference between the time the front of a vehicle arrives at a point on the highway and the time the front of the next vehicle arrives at that same point. Time headway is usually expressed in seconds.

vii. \( \text{Space headway} (d) \) is the distance between the front of a vehicle and the front of the following vehicle and is usually expressed in feet.

In the course of the research, data in Table were collected on the three most strategic spots (Kwangila, Dan-Magaji and Mararba-Jos) of Zaria-Kaduna road to test both Greenshields and Greenberg macroscopic models.

**Flow-Density Relationships**

Generally, the equations relating flow, density, and space mean speed is given as:

\[
\begin{align*}
q &= k \times \bar{u} \\
\text{Flow} &= \text{Density} \times \text{Space Mean Speed}
\end{align*}
\]

Also, other relationships that exist among the traffic-flow variables are by:

\[
\begin{align*}
speed &= \frac{flow}{headway space} \\
\text{speed} &= \frac{q}{d}
\end{align*}
\]

where \( d \) = Average Space headway = \( \frac{1}{k} \)

\[
\text{Density} = \frac{Flow}{Average travel time for unit distance i.e.}
\]

\[
\begin{align*}
h &= \frac{q^2}{d^2} \\
\text{Density} &= \frac{Flow}{Average space headway}
\end{align*}
\]

\[
\begin{align*}
T &= \frac{1}{\bar{u}} = \frac{d}{\bar{u}_s} \\
\text{Average time headway} &= \frac{space mean speed \times \text{Average time headway}}{d}
\end{align*}
\]

\[
\begin{align*}
F &= \frac{1}{d} \\
\text{Average travel time for distance unit} &= \text{Average space headway}
\end{align*}
\]

**Traffic Flow Mathematical Modelling**

The relationships describing traffic flow can be classified into two general categories-macroscopic and microscopic depending on the approach used in the development of these relationships. The macroscopic approach considers flow density relationships whereas the microscopic approach considers spacings between vehicles and speeds of individual vehicles.

**Macroscopic Approach**

The macroscopic approach considers traffic streams and develops algorithms that relate the flow to the density and space mean speeds. The two most commonly used macroscopic models are the Greenshields and Greenberg models.

**Greenshields Model:** Greenshields carried out one of the earliest recorded works in which he studied the relationship between speed and density. He hypothesized that a linear relationship
existed between speed and density which he expressed as
\[ \bar{a}_2 = u_f - \frac{a_f}{b_f} \]  
Equation (4) gives rise to
\[ \bar{a}_2 = \frac{u_f}{a_f} \]  
Substituting for \( \bar{a}_2 = \frac{u_f}{a_f} \) into equation (10) gives
\[ \bar{a}_2 = u_f - \frac{a_f}{b_f} \]  
And substituting for \( \bar{a}_2 = \frac{u_f}{a_f} \) into equation (10) gives
\[ u_f = a_f - \frac{a_f^2}{b_f^2} \]  
From equation (12),
\[ q \]  
is differentiated with respect to \( \bar{a}_2 \) which gives
\[ 2\bar{a}_2 = u_f - 2a_f \frac{b_f}{b_f^2} \]  
Rearranging,
\[ \frac{dq}{da_2} = u_f - 2a_f \frac{b_f}{b_f^2} = b_f - 2a_f \frac{b_f}{b_f^2} \]  
For maximum flow, \( \frac{dq}{da_2} = 0 \), and equation (14) becomes
\[ u_f = \frac{b_f}{2} \]  
which shows that space mean speed \( u_f \) at which the volume is maximum is equal to half the free mean speed. From equation (13), differentiating \( q \) with respect to \( k \) gives:
\[ \frac{dq}{dk} = u_f - 2a_f \frac{b_f}{b_f^2} \]  
For maximum flow, \( \frac{dq}{dk} = 0 \), and equation (16) becomes
\[ k = \frac{b_f}{2} \]  
Hence, at the maximum flow, the density \( k \) is half the jam density. The maximum flow for the Greenshields relationship can therefore be obtained from equation (4), (15) and (17) i.e.
\[ q_{\text{max}} = \frac{b_f u_f}{4} \]  

Greenberg Model

Several researchers have used the analogy of fluid flow to develop macroscopic relationships for traffic flow. One of the major contributions using the fluid-flow analogy was developed by Greenberg in the form:
\[ \bar{a}_2 = c \ln \frac{b_f}{k} \]  
Multiplying through by \( k \);
\[ \bar{a}_2 = c \ln \frac{b_f}{k} = q \]  
And differentiating \( q \) with respect to \( k \) gives
\[ \frac{dq}{dk} = c \ln \frac{b_f}{k} = c \]  
For maximum flow,
\[ \frac{dq}{dk} = 0, \text{ and equation (20) becomes} \]
\[ \ln \frac{b_f}{k} = 1 \]  
Substituting 1 for \( (k/k_c) \) into equation 19 gives
\[ a_f = c \]  
Thus, the value of \( c \) is the speed at maximum flow.

Fitting the Macroscopic Model

The most common method of approach is regression analysis. This is done by minimizing the squares of the differences between the observed and expected values of a dependent variable. When the dependent variable is linearly related to the independent variable, the process is known as linear regression analysis. When the relationship is with two or more independent variables, the process is known as multiple linear regression analysis.

If a dependent variable \( y \) and an independent variable \( x \) are related by an estimated regression function, then
\[ y = a + bx \]  
where the constants
\[ a = \frac{1}{n} \sum_{i=1}^{n} x_i - \frac{1}{n} \sum_{i=1}^{n} y_i \]  
\[ b = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n} \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n} (\sum_{i=1}^{n} x_i)^2} \]  
Where \( n \) = number of sets of observations; \( x_i = \text{ith} \) observation for \( x \); \( y_i = \text{ith} \) observation for \( y \)

A measure commonly used to determine the suitability of an estimated regression function is the coefficient of determination (or square of the estimated correlation coefficient) \( R^2 \), which is given by
\[ R^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]  
where \( Y_i \) is the value of the dependent variable as computed from the regression equations.

The closer \( R^2 \) is to 1, the better the regression fits.

RESULTS

In the analysis, the dependent variable, speed is linearly related to the independent variable, density, therefore, linear regression analysis applied. The various parameters in result of macroscopic modelling of the data in Table 2 are as explained before, while the plots of
the results are in Figures 1, 2 and 3 for results of spots 1, 2 and 3. The basic parameters of Greenshields’ model are free flow speed and density. The values for free flow speed for the three spots \( (u_f) \) are respectively 108.8, 111.1995 and 115.3875mph. Also, their jam densities \( (k_f) \) 241, 201 and 205vpm at respective maximum flow \( (q_{\text{max}}) \) 6531, 5588 and 5914vpm. Greenberg’s model is more sensitive to predicting the jam densities. The values of jam density \( (k_f) \), maximum flow \( (q_{\text{max}}) \) and speed at maximum flow \( (c) \) on the three spots are 550, 366 and 367vpm; 6809, 5368 and 5700vpm; and 33.6595, 39.8796 and 42.2232mph.

<table>
<thead>
<tr>
<th>Spot 1 (Kwangila)</th>
<th>Spot 2 (Dan-Magaji)</th>
<th>Spot 3 (Mararba-Jos)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed ( u ) (mph)</td>
<td>Density ( k ) (vpm)</td>
<td>Speed ( u ) (mph)</td>
</tr>
<tr>
<td>27.7</td>
<td>175</td>
<td>27.7</td>
</tr>
<tr>
<td>33.9</td>
<td>166</td>
<td>33.9</td>
</tr>
<tr>
<td>40.1</td>
<td>151</td>
<td>40.1</td>
</tr>
<tr>
<td>46.3</td>
<td>139</td>
<td>46.3</td>
</tr>
<tr>
<td>52.5</td>
<td>127</td>
<td>52.5</td>
</tr>
<tr>
<td>58.7</td>
<td>113</td>
<td>58.7</td>
</tr>
<tr>
<td>64.9</td>
<td>102</td>
<td>64.9</td>
</tr>
<tr>
<td>71.1</td>
<td>88</td>
<td>71.1</td>
</tr>
<tr>
<td>77.4</td>
<td>62</td>
<td>77.4</td>
</tr>
<tr>
<td>83.6</td>
<td>43</td>
<td>83.6</td>
</tr>
<tr>
<td>89.8</td>
<td>38</td>
<td>89.8</td>
</tr>
<tr>
<td>96.0</td>
<td>31</td>
<td>96.0</td>
</tr>
<tr>
<td>102.2</td>
<td>22</td>
<td>102.2</td>
</tr>
</tbody>
</table>

### Table 2: This table shows parameters of macroscopic modeling of speed-density data

<table>
<thead>
<tr>
<th>Traffic Parameters</th>
<th>Spot 1</th>
<th>Spot 2</th>
<th>Spot 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>108.4</td>
<td>111.1995</td>
<td>115.3875</td>
</tr>
<tr>
<td>b</td>
<td>-0.44956</td>
<td>-0.55434</td>
<td>-0.56349</td>
</tr>
<tr>
<td>( u_f ) (mph)</td>
<td>108.4</td>
<td>-</td>
<td>111.1995</td>
</tr>
<tr>
<td>( k_f ) (vpm)</td>
<td>0.44956</td>
<td>0.55434</td>
<td>0.56349</td>
</tr>
<tr>
<td>( q_{\text{max}} ) ( (c) ) (mph)</td>
<td>241</td>
<td>201</td>
<td>205</td>
</tr>
<tr>
<td>( k_f ) (vpm)</td>
<td>54.2</td>
<td>55.6</td>
<td>57.7</td>
</tr>
<tr>
<td>( q_{\text{max}} ) ( (c) ) (mph)</td>
<td>120.5</td>
<td>100.5</td>
<td>102.5</td>
</tr>
<tr>
<td>( k_f ) (vpm)</td>
<td>-</td>
<td>33.6595</td>
<td>39.8796</td>
</tr>
<tr>
<td>( q_{\text{max}} ) ( (c) ) (mph)</td>
<td>-</td>
<td>235.4298</td>
<td>-</td>
</tr>
<tr>
<td>( k_f ) (vpm)</td>
<td>6531</td>
<td>5588</td>
<td>5368</td>
</tr>
<tr>
<td>( q_{\text{max}} ) ( (c) ) (mph)</td>
<td>6809</td>
<td>5700</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>( R^2 = 0.989 )</td>
<td>( 0.984 )</td>
<td>( 0.992 )</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### DISCUSSION

The values of regression analysis \( (R^2) \) showed that Greenberg’s model fits the data better than Greenshield’s model in all the three spots. However, the drawback of Greenberg’s model is that as density tends towards zero, speed tends to infinity showing that it is ineffective at lower densities. But the problem can be solved using Greenshield’s model. It is clearly evident in both macroscopic and microscopic models illustrated that the values of max flow rates \( (q_{\text{max}}) \) are not abating or
dissipating but rather maintaining wavering trends due to increasing traffic that affects the capacity of the highway. This is owing to the fact that the rate of traffic dissipation is fairly equal to the attraction into stream; a consequent rising number of automobile and bad road condition in the study area.

**CONCLUSION**

Despite the significant efforts being made to reduce the number of fatal casualties in road traffic by identifying and rectifying flaws in road design along with improvement in car safety and traffic management, the increasing growth of traffic density has been a major challenge for road traffic engineers not only for dealing with the lasting congestion problem but also for addressing environmental pollution and road traffic safety related issues.

The usage of the developed models will enhance prediction of congested traffic and jams situations and elaborates recommendations on avoiding and mitigating the problems. Furthermore, the models will enhance analysis of the influence of geometric road conditions, road standards and handling regimes on the capacity of traffic networks and to solve many other problems of traffic flow dynamics. In contrast, the models will not be effective in predicting how changes in design might influence the probability of collisions as well as provide framework on how changes in the roadside environment affects driver behaviour within the travel way.

**REFERENCES**


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Nil

CONFLICT OF INTEREST
No conflicts of interests were declared by authors.

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