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Incorporation of Advanced Second Moment Reliability Assessment Method into Assessment and Design

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ABSTRACT [ENGLISH/ANGLAIS]

The concept of using the advance second moment reliability assessment method (ASMRAM) for structural analysis and assessment is presented. This is a full probability approach to structural assessment which gives a deterministic result and save parameters considering all the possible variability in structural assessment. The load, material and geometrical properties of a structure which determine the performance function of the system are treated as random quantities with assumed practical probability distributions. A concrete beam of fixed cross section is analyzed and assessed within the context of possible variables to appreciate the effectiveness of ASMRAM in comparison to the traditional deterministic partial safety factor design. The intent of the work is realized through user friendly computer modules developed in FORTRAN programming language.

Keywords: Reliability, Probability analysis, structural uncertainties, ASMRAM, FORTRAN

RÉSUMÉ [FRANÇAIS/FRENCH]

Le concept d'utilisation de la deuxième méthode d'évaluation préalable instant la fiabilité (ASMRAM) pour l'analyse structurelle et de l'évaluation est présenté. Il s'agit d'une approche probabiliste complète à l'évaluation structurelle qui donne un résultat déterministe et sauvegarder les paramètres en considération toutes les variations possibles dans l'évaluation structurale. Les propriétés de charge, matérielles et géométriques d'une structure qui déterminent la fonction de performance du système sont traités comme des quantités aléatoires avec des distributions de probabilités supposées pratiques. Une poutre en béton de section fixe est analysée et évaluée dans le contexte de variables possibles pour apprécier l'efficacité de ASMRAM par rapport à la traditionnelle conception déterministe de la sécurité partielle des facteurs. Le but du travail est réalisé à travers des modules d'utilisateurs d'ordinateurs respectueux développés en langage de programmation Fortran.

Mots-clés: Fiabilité, analyse de probabilité, les incertitudes structurelles, ASMRAM, FORTRAN

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INTRODUCTION

Traditionally, conventional design practice covers the possibility of actual loads exceeding assumed load by the application of arbitrary safety factors. But phenomena in real world have shown that structural problems may be treated as often non deterministic. Moreover, design and construction are executed despite imperfect knowledge and information [1]. Deterministic safety factors are usually incorporated into design routines to cater for lack of information or imperfection. Consequently, the luxury of selecting the worst possible load condition and designing for that load is no longer economically feasible under a broad spectrum of load conditions [2]. The application of Advance Second Moment Reliability Assessment Method will offer a meaningful solution.

MATERIALS AND METHODS

Deterministic Design Concept

The conventional deterministic design practice used for the current European standards are based on the semi-probabilistic approach to structural reliability assessment with the help of Partial Safety Factors (PSF) to cover the possibility of actual loads exceeding assumed load. The assumed load also called the design load is always a factored load which is directly related to an arbitrary factored material resistance to cater for uncertainty for lack of information or imperfection. The possibility of the actual load exceeding the assumed load is considered to be directly related to the uncertainties inherent in material properties and loading upon the structure.

In view of this inherent uncertainty incorporated with structural performance, the conventional deterministic design approach can be writing as:

$$v_R R \geq v_L L \quad (1)$$

Where R and L = material resistance and load effect respectivelyv_R and v_L are arbitrary safety factors for materialresistance and load effect respectively

Advance Second Moment Reliability Assessment Method

Reliability concept uses a function, Z(X), which determines the performance of the structural system. Thus it is also known as the performance function of the system. Therefore the performance function of the structural system is always defined as

$$Z(X) = R(X) - L(X) \quad (2)$$

Where $R(X) and L(X) are material resistances and load effects which depend on the random variable X$.

At the extreme point of failure (or the limit sate),$R(X) = L(X) and Z(X) = 0$. Thus, the point of interest can be defined as $Z(X) = 0$. When $Z(X) < 0$, the element is in the failure state, and when $Z(X) > 0$, it is in the survival state.

If the joint probability density function for the basic random variables X_i's is $f_{x_1,x_2,\ldots x_n}(x_1, x_2, \ldots x_n)$, then the possibility of failure known as the failure probability P_f of the element can be given by the integral

$$P_f = \int \ldots \int f_{x_1,x_2,\ldots,x_n}(x_1, x_2, \ldots x_n)\, d_{x_1,x_2} \ldots d_{x_n} \quad (3)$$

Where the integration is performed over the region in which $Z(X) < 0$.

In general, the joint probability density function is unknown, and the integral is a formidable task. [3]. For practical purposes, alternate method of evaluating P_f is necessary. The possibility of survival (existence) of the structure known as the RELIABILITY is assessed as one minus the failure probability.

Instead of integrating *Eq. 3* directly, the performance function $Z(X)$ in *Eq. 2* can be expanded using Taylor series about the mean value of Xs and then truncated at the linear term. Therefore, according to (Assakkaf, 2004), the first order approximation for the mean and variance of the performance function are as follows:

$$\mu_z \approx Z\left(\mu_{x_1,x_2,\ldots,x_n}\right) \quad (4)$$

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial z}{\partial x_i}\right)\left(\frac{\partial z}{\partial x_j}\right) Cov(X_i, X_j) \quad (5a)$$

Where $\mu = mean\ of\ random\ variable;$ $\mu_Z = mean\ of\ Z; \sigma_Z^2 = variance\ of\ Z$; $Cov(X_i, X_j) =$ $Covariance\ of\ X_1\ and\ X_2; \frac{\partial z}{\partial x_i}\ and\ \left(\frac{\partial z}{\partial x_j}\right)\ are\ partial$ $derivative\ evaluated\ at\ the\ mean\ of\ random\ variable$

For uncorrelated random variables, the variance can be expressed as

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\sigma_{X_i}^2\left(\frac{\partial z}{\partial x_i}\right)^2 \quad (5b)$$

The reliability index β can be computed from:

$$\beta = \frac{\mu_Z}{\sigma_Z} \quad (6)$$

$$P_f = 1 - \Phi(\beta) \quad (7)$$

The equation above holds only when Z is linear and normally distributed [4]

For nonlinear performance functions, the Taylor series expansion of $Z(X)$ is linearized at some point on the failure surface referred to as the design point or checking point or the most probable failure point rather than at the mean.

Assuming the performance function for material resistance and load effect is given by $Z(X_i) = R(X_i) - L(X_i)$, where X_i are uncorrelated random variables; the following transformation to reduced or normalized coordinates can be used:

$$Y_R = \frac{R(X_i) - \mu_{X_R}}{\sigma_{X_R}} \text{ and } Y_L = \frac{L(X_i) - \mu_{X_L}}{\sigma_{X_L}} \quad (8a)$$

Where $\mu_{Y_i} = 0\ and\ \sigma_{Y_i} = 1$

At the failure surface, $R(X_i) - L(X_i) = 0\ or\ R(X_i) = L(X_i)$

This implies that

$$Y_L = \frac{\sigma_{X_R}}{\sigma_{X_L}}Y_R + \frac{\mu_{X_R} - \mu_{X_L}}{\sigma_{X_L}}\ or\ Y_L\sigma_{X_L} - Y_R\sigma_{X_R} - (\mu_{X_R} - \mu_{X_L}) = 0 \quad (8b)$$

Eq. 8b is equation of a straight line.

Considering Figure 1(b) and dividing equation8b through by $\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2};$ we have:

$$\frac{Y_L\sigma_{X_L}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} - \frac{Y_R\sigma_{X_R}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} - \frac{(\mu_{X_R} - \mu_{X_L})}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} = 0$$

This implies that $Y_L\alpha_{X_L} - Y_R\alpha_{X_R} - P = 0$

$$\alpha_{X_L} = \frac{\sigma X_L}{\sqrt{\sigma^2_{X_R}+\sigma^2_{X_L}}};\ \alpha_{X_R} = \frac{\sigma X_R}{\sqrt{\sigma^2_{X_R}+\sigma^2_{X_L}}};$$

Where

$$P = \frac{(\mu_{X_R} - \mu_{X_L})}{\sqrt{\sigma^2_{X_R} + \sigma^2_{X_L}}};\ \alpha_{X_i} = directional cosine of the$$

perpendicular distance from the origin and

P = *perpendicular distance form the origin*

The perpendicular distance of the linearly normalized performance function from the origin which is also the shortest distance from the origin, is a measure of the possible design point (failure point) given by $Z(X_i^*) = (X_1^*, X_2^*, \ldots, X_n^*)$ from the origin (mean). This indicates that P is a measure of the system reliability, thus;

$$P = \beta = \frac{\mu X_R - \mu X_L}{\sqrt{\sigma^2_{X_R}+\sigma^2_{X_L}}} \tag{9}$$

The concept of perpendicular and shortest distance applies to linear performance function; while for nonlinear performance function only the shortest distance dictate the possible failure point, as shown in Figure 1(c).

Eq. 9 can only be use for a two variant performance function in a linear plane. But for a multi-variant performance function on an hyperplane, β is always given as in Eq. 6, that is;

$$\beta = \frac{\mu_Z}{\sigma_Z}$$

but from 5b, $\sigma_Z^2 \approx \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial z}{\partial x_i}\right)^2$, thus;

$$\beta = \frac{\mu_Z}{\left\{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2\right\}^{1/2}}$$

$$or\ \mu_Z = \beta\left\{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2\right\}^{1/2}$$

$$\mu_Z = \beta\left\{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2\right\}^{1/2} \times \frac{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2}{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2}$$

$$\mu_Z = \beta \times \frac{\sum_{i=1}^n \sigma_{X_i}\left(\frac{\partial Z}{\partial X_i}\right) \times \sum_{i=1}^n \sigma_{X_i}\left(\frac{\partial Z}{\partial X_i}\right)}{\left\{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2\right\}^{1/2}}$$

Thus,

$$\mu_Z = \beta\alpha_{X_i} \sum_{i=1}^n \sigma_{X_i}\left(\frac{\partial Z}{\partial X_i}\right) \tag{10}$$

$$where\ \alpha_{X_i} = \frac{\sum_{i=1}^n \sigma X_i \left(\frac{\partial Z}{\partial X_i}\right)}{\left\{\sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2\right\}^{1/2}} \tag{11}$$

α_{X_i} = *directional cosine on an hyperplane*

On the failure surface, at the possible failure point

$$Z(X_1^*, X_2^*, \ldots, X_n^*) = 0 \tag{12}$$

But also, from equation 10;

$$\mu_Z - \beta\alpha_{X_i} \sum_{i=1}^n \sigma_{X_i}\left(\frac{\partial Z}{\partial X_i}\right) = 0$$

Therefore,

$$Z(X_1^*, X_2^*, \ldots, X_n^*) = \mu_Z - \beta\alpha_{X_i} \sum_{i=1}^n \sigma_{X_i}\left(\frac{\partial Z}{\partial X_i}\right)$$

Thus, the corresponding failure points (design point) X_i^*, can be written as:

$$X_i^* = \mu_{X_i} - \beta\alpha_{X_i}\sigma_{X_i} \tag{13}$$

Also;

$$\frac{X_i^*}{\mu X_i} = \gamma_{X_i} = 1 - \beta\alpha_{X_i}\vartheta_{X_i} \tag{14}$$

Where γ_{X_i} = *safety factor of variable* X_i

$\vartheta_{X_i} = \frac{\mu_{X_i}}{\sigma_{X_i}}$ = *coefficient of variation of variable* X_i

The partial derivatives are evaluated at the design point, X_i^*. The reliability index β and the design point, $(X_1^*, X_2^*, \ldots, X_n^*)$ can be determined by solving the nonlinear equations 11, 12, and 13 iteratively for β [5]. Also, partial safety factors, γ, for individual variable can be calculated from 14.

Generally, partial safety factors take on values larger than 1 for loads, and less than 1 for strengths. Equations 7 can be used to compute the probability of failure, P_f corresponding to the deterministic design point $Z\ (X_1^*, X_2^*, \ldots, X_n^*)$.

The directional cosines are considered as measure of the importance of the corresponding random variable in determining the reliability index, β. However, the above formulation is limited to normally distributed random variables.

For random variable X which is not normally distributed must be transformed to an equivalent normally distributed random variable.The parameters of the equivalent normal distribution are $\mu_{X_i}^N$ *and* $\sigma_{X_i}^N$.

Assakkaf (2004) stated that these parameters can be estimated by imposing two conditions. Viz;

$$\Phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_i(X_i^*) \qquad (15a)$$

And

$$\phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_i(X_i^*) \qquad (15b)$$

Where

F_i = non normal cumulative distribution function

f_i = non normal probability density function

Φ = cumulative distribution function of the standard normal variate

ϕ = probability density function of the standard normal variate

The standard deviation and mean of equivalent normal distributions are given by

$$\sigma_{x_i}^N = \frac{\phi(\Phi^{-1}[F_i(X_i^*)])}{f_i(X_i^*)} \qquad (16a)$$

$$\mu_{X_i}^N = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_{X_i}^N \qquad (16b)$$

Once $\sigma_{X_i}^N$ and $\mu_{X_i}^N$, are determined for each random variable, β can be solved following the same procedure for normal distribution.

RESULTS

Illustrations

Consider a case of a simply supported concrete beam whose failure is defined by crushing of the concrete before yielding of the reinforcement. The design data are as given in Table 1.

Partial factor safety design (EURO CODE 2)

From EUROCODE 2, the design resistance moment, $M_R \cong 0.156 f_{cu} bd^2$ or

$$M_R \cong 0.234\left(\frac{1}{\gamma_m}\right) f_{cu} bd^2$$

Where;

γ_m = partialsafetyforconcretematerial strenghtunderflexuralaction = 1.5

f_{cu} = concretecharacteristicstrenght

b, d = breathanddepthofbeamcross sectionrespectively

The assumed design load, D_L, is given as

$$D_L = \gamma_D G_k + \gamma_l Q_k$$

And the ultimate applied moment, M_L, for a uniformly distributed load Is given by

$$M_L = \frac{(\gamma_D G_k + \gamma_l Q_k)L^2}{8}$$

Where γ_D, γ_l = deadandimposeloadsafety factorerespectively

G_k, Q_k = characteristicdeadandimposeload respectively. Using the table of data given above, $M_R = 80.73 KNm$ and $M_L = 78.75 KNm$.

Thus, for ultimate limit state considering failure, $M_R > M_L$. The section is safe under the extreme condition.

ADVANCE SECOND MOMENT RELIABILITY ASSESSMENT METHOD

The resistance moment, M_R, and applied moment, M_L, can be model as:

$M_R = 0.234 f_{cu} bd^2$ and $M_L = \frac{L^2}{8}(G_K + Q_K)$,

so that the performance function, $Z(X_i)$, given that $f_{cu} = X_1, G_K = X_2$ and $Q_K = X_3$, is defined by:

$$Z(X_i) = 0.234 X_1 bd^2 - \frac{L^2}{8}(G_K + Q_K).$$

where b, d and L, are constants as given in the Table 1 below. This implies that;

$Z(X_i) = 4.8438 X_1 - 3.125(G_K + Q_K)$. This is a linear function with constant partial derivative with respect to individual variable as given bellow;

$$\frac{\partial Z(X_i)}{\partial X_1} = 4.8438, \frac{\partial Z(X_i)}{\partial X_2} = -3.125 \text{ and } \frac{\partial Z(X_i)}{\partial X_3} = -3.125$$

Using equations 11, 12 and 13, the possibility of existence of the beam defined by reliability index, β, is given by

$$\beta = \frac{A\mu_{X_1} - B(\mu_{X_2} - \mu_{X_3})}{A\alpha_{X_1}\sigma_{X_1} - B(\alpha_{X_2}\sigma_{X_2} + \alpha_{X_3}\sigma_{X_3})}$$

Where; $A = 4.8438$ and $B = 3.125$

With a constant partial derivative, this implies that, β, is governed by σ_{X_i}, which measures the degree of inconsistency inherent within the variables. This is given in Table 2 – Table 6 below.

Considering Table 2 to Table 6, it was observed that the degree of entropy measured in terms of σ_{X_i} has a profound effect on the measure of uncertainty inherent within the considered variable. The more scatter the variable is the higher its entropy and the higher the factor of safety. Therefore, in conditions of high variability, the conventional deterministic approach may be unreliable as a result of either too low on load effect or too high on material resistance. Thus, the ASMRAM will be an effective approach to some degree of measured accuracy considering possible variables.

DISCUSSION

The ASMRAM assessment is also an appreciable technique in assessing the possibility of existence of the structure within an assumed degree of variability. As illustrated in the Tables the possibility of existence which can also be assessed as the failure probability is a measure of the degree of variability. Thus, the higher the entropy, the more the inherent risk and the higher the possibility of failure. Therefore at any point, under any given condition, ASMRAM can be used for structural assessment with incorporated degree of failure as possibility of existence. Also, ASMRAM in combination with reasonable computation can be used to analyze and assess the variability of a structure adaptable to new knowledge, developments in theories, change in priority, and demand of structural utility. ASRAM method can be incorporated into design routines to cater for lack of information or imperfection. There are several relatively new circumstances which make this philosophy necessary; such as the appearance of a vast array of new structural materials due to structural complexities involving operations near limit state.

Table 1: This table shows design data.

Data	Variable(X_i)	Characteristic Value,(μ_{Xi})	StandardDéviation, (σ_{Xi})	Partial SafetyFactor,(γ)	Type of distribution
f_{cu}	X_1	$25N/mm^2$	$4-7$	1.5	Normal Distribution
G_K	X_2	$10KN/m$	$0.25-2.5$	1.4	Normal Distribution
Q_K	X_3	$7KN/m$	$0.35-3.5$	1.6	Normal Distribution
B	Constant	$230mm$	Constant	Constant	Constant
D	Constant	$300mm$	Constant	Constant	Constant
L	Constant	$5000mm$	Constant	Constant	Constant

Table 2: This table shows parameters of variables at $\beta = 3.4997$, $Pf = 0.0002$

variable	μ_{Xi}	σ_{Xi}	$\partial Z/\partial X_{xi}$	$(\partial Z/\partial X_{xi})\ \sigma_{xi}$	α_{xi}	X_i^*	γ_{Xi}
X_1	25	4.000E0	4.8438	19.3752E0	9.9796E-1	11.0348	0.4400
X_2	10	2.500E-1	-3.1250	-7.8125E-1	-4.023E-2	10.0350	1.0035
X_3	7	3.500E-1	-3.1250	-1.0938E0	-5.632E-2	07.0690	1.0098

Table 3: This table shows parameters of variables at $\beta = 2.7399$, $Pf = 0.0031$

variable	μ_{Xi}	σ_{Xi}	$\partial Z/\partial X_{xi}$	$(\partial Z/\partial X_{xi})\ \sigma_{xi}$	α_{xi}	X_i^*	γ_{Xi}
X_1	25	5.000E0	4.8438	2.4219E1	9.7620E-1	11.6270	0.4650
X_2	10	1.000E0	-3.1250	-3.1250E0	-1.2596E-1	10.3450	1.0345
X_3	7	1.400E0	-3.1250	-4.3750E0	-1.7635E-1	07.6760	1.0966

Table 4: This table shows parameters of variables at $\beta = 2.2250$, $Pf = 0.0132$

variable	μ_{Xi}	σ_{Xi}	$\partial Z/\partial X_{xi}$	$(\partial Z/\partial X_{xi})\ \sigma_{xi}$	α_{xi}	X_i^*	γ_{Xi}
X_1	25	6.000E0	4.8438	2.9063E1	9.5140E-1	12.2988	0.4920
X_2	10	1.750E0	-3.1250	-5.4688E0	-1.7900E-1	10.6970	1.0697
X_3	7	2.450E0	-3.1250	-7.6563E0	-2.5063E-1	08.3660	1.1950

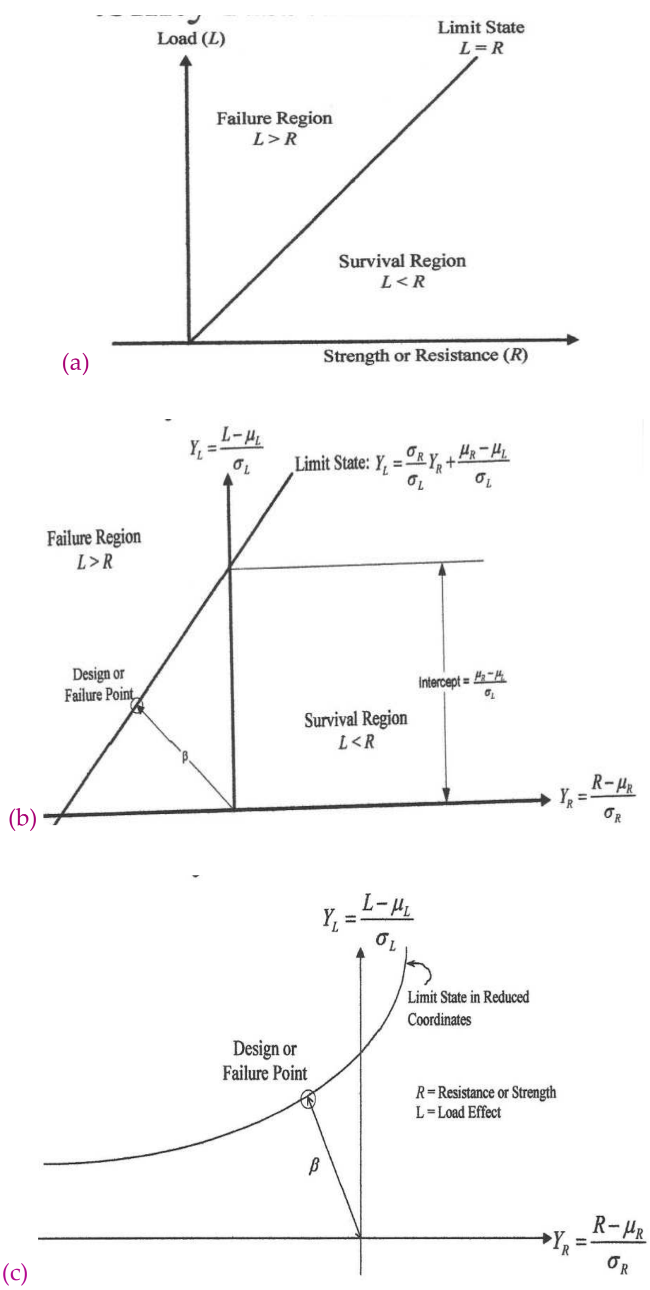
Table 5: This table shows parameters of variables at $\beta = 2.0294$, $Pf = 0.0217$

variable	μ_{xi}	σ_{Xi}	$\partial Z/\partial X_{xi}$	$(\partial Z/\partial X_{xi})\ \sigma_{xi}$	α_{xi}	X_i^*	γ_{Xi}
X_1	25	6.500E0	4.8438	3.1485E 1	9.4000E-1	12.6000	0.5040
X_2	10	2.125E-1	-3.1250	-6.6406E0	-1.9826E-1	10.8550	1.0855
X_3	7	2.975E-1	-3.1250	-9.2969E0	-2.7757E-1	08.6758	1.2394

Table 6: This table shows parameters of variables at $\beta = 1.8636, Pf = 0.0314$

variable	μ_{Xi}	σ_{Xi}	$\partial Z/\partial X_{Xi}$	$(\partial Z/\partial X_{xi})\ \sigma_{Xi}$	α_{Xi}	X_i^*	γ_{Xi}
X_1	25	7.000E0	4.8438	3.3907E1	9.2960E-1	12.8730	0.5149
X_2	10	2.500E0	-3.1250	-7.8125E0	-2.1420E-1	10.9980	1.0998
X_3	7	3.500E0	-3.1250	-1.0938E1	-2.9987E-1	08.9559	1.2794

Figure 1: This figure shows (a) performance function for a linear, two normally distributed random variable case. (b)Performance function for a linear, two random variable case in normalized coordinates. (c) Performance function for a nonlinear, two random variable case in a normalized coordinates.



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CONFLICT OF INTEREST

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