

Original Article

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## Machinery Replacement Problems Model

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## ABSTRACT [ENGLISH/ANGLAIS]

The effect of deterioration on resale value is examined in this study. Previous works on machinery replacement lay emphasis on the minimization of total, maintenance and operating costs and the maximization of profit without recourse to the effect of deterioration on the resale value. Some models exclude resale value in the build-up of cost. In other models, values of deterioration are assumed or at best determined by methods that are highly subjective and sometimes expensive like the popular failure analysis. In this work, values of deterioration are generated as random numbers using the Monte Carlo simulation under the uniform probability distribution. Dynamic programming enumeration process is adopted as the solution technique. The model is calibrated and the results are verified with field data from different industries. Finally, the results of the model are compared with those of existing models. Basically, the results show that with regular and timely corrective maintenance the optimal replacement dates are between 4 and 6 years for construction machines, 16 and 20 years for pharmaceutical machines, 13 and 17 years for plastic machines and 6 and 9 years for transport vehicles. The results also show that the model is reliable, operational and simple in application.

**Keywords:** Replacement model, deterioration/salvage value, dynamic programming

## RÉSUMÉ [FRANÇAIS/FRENCH]

L'effet de la détérioration sur la valeur de revente est examiné dans cette étude. Des travaux antérieurs sur le remplacement des machines mettent l'accent sur la minimisation des coûts totaux, de maintenance et d'exploitation et la maximisation du profit sans avoir recours à l'effet de la détérioration de la valeur de revente. Certains modèles d'exclure la valeur de revente de l'accumulation des coûts. Dans d'autres modèles, les valeurs de détérioration sont supposées ou au mieux déterminée par des méthodes qui sont très subjectifs et parfois coûteux, comme l'analyse des défaillances populaire. Dans ce travail, les valeurs de détérioration sont générés sous forme de nombres aléatoires en utilisant la simulation de Monte-Carlo sous la distribution de probabilité uniforme. Dynamique processus de recensement de programmation est adopté comme solution de la technique. Le modèle est calibré et les résultats sont vérifiées avec les données de terrain provenant de différentes industries. Enfin, les résultats du modèle sont comparées avec celles des modèles existants. Fondamentalement, les résultats montrent que, avec la maintenance corrective régulière et en temps opportun les dates de remplacement optimale se situe entre 4 et 6 ans pour les machines de construction, 16 et 20 ans pour les machines pharmaceutiques, 13 et 17 ans pour les machines en plastique, 6 et 9 ans pour les véhicules de transport. Les résultats montrent également que le modèle est fiable, opérationnelle et simple dans son application.

**Mots-clés:** Modèle de remplacement, la détérioration / la valeur de récupération, la programmation dynamique

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## INTRODUCTION

Replacement of machinery is not widely discussed in Engineering Literatures. Yet it is a common occurrence in industries requiring the use of plant and equipment for the production of goods and services. Even where it is discussed, the emphasis is usually placed on minimization of total, maintenance and operating costs as well as the maximization of profit without recourse to the uncertainty resulting from the method of determining deterioration. Cognizance of the effect of deterioration on the resale value of equipment and indeed on machinery replacement date is yet to be fully appreciated. Values of deterioration are usually assumed

or at best determined by methods that are highly subjective and sometimes expensive like the popular failure analysis [1]. Some replacement models exclude resale value in the build-up of cost. Yet it is the value that is directly affected by deterioration. This missing link will be supplied by this study. The six main groups of machinery replacement models identified in the study include the New operating machine replacement models [2] model is an example of the group), the Old operating machine replacement models [3], the Failed machine replacement models [4], the Low cost items replacement models (Group replacement) [5], the Planning horizon models (Sequence of replacement) [6] and the

Simultaneous maintenance and replacement models [7]. The limitations of these models include complexity due to the relaxing of certain assumptions during formulation, non-incorporation of economic parameters, infinite solutions, imposition of heavy penalty for failures before replacement and the unrealistic nature of some of the models hence the difficulties experienced during implementation. The objective of this paper is to formulate a cost minimization replacement model with emphasis on the effect of deterioration on the salvage value, incorporating relevant economic parameters and employing the dynamic programming technique as the solution method.

## METHODOLOGY

### Analytical Process

Maintenance costs and salvage values are obtained for the front-end Loader, CAT 140H Motor Grader, Capsule filling machine, Injection moulding machine, Mercedes Benz Express bus and Hiace Mini bus. Random numbers are generated and converted under the uniform probability distribution to get values for deterioration. Then the model is calibrated and the dynamic programming technique [8] is applied to obtain the measured and predicted total costs  $K(T)$  which reach a minimum corresponding to the optimal replacement date. Finally graphs of measured and predicted total costs against time are plotted. The correlation coefficient,  $r_c$  between the measured and predicted total costs is derived to ascertain the strength and linear relationship as well as the degree of dependence of the two variables.

### Experimental Process

Field data (Maintenance costs and salvage values) are obtained from the following companies,

- Mothercat Limited, Mando, Kaduna
- Juhel Nigeria Limited, Emene, Enugu
- Innoson Technical and Industrial Company Nigeria Ltd, Emene Enugu
- Associated Bus Company PLC, Owerri, Imo State.

The field data obtained from these companies is used to verify the reliability of the results from the model. Results from the model are compared with the results obtained from the model by Walker [9] using field data from Associated Bus Company PLC.

### Model Formulation

The model has three components (centres) namely the purchase cost,  $Q$ , the maintenance cost,  $B(t)$  and the

Salvage (resale) value,  $S(t)$ . The cost components are discounted to give the present value of the total cost.

Cost minimization model by Ekeocha [10] is given by

$$K(T) = \frac{1}{R} [Q + B(t)R - S(t)R^t] \quad (1)$$

$$K(T) = \frac{1}{R} [Q + (b_1 + b_2 t)R - Q(1 - d)^t R^t] \quad (2)$$

The objective is to find  $T$  that minimizes total cost,  $K(T)$ .

### Assumptions

- The purchase cost,  $Q$  is constant and may include the cost of installation and other incidental cost before the machine is put into use.
- The maintenance cost,  $B(t)$  is non-linear as it increases disproportionately whenever there is refurbishment.
- The coefficient  $b_1$  represents the constant component of the maintenance cost (routine maintenance)
- The coefficient  $b_2$  represents the increase per period component of the time – dependent maintenance cost. It takes care of downtime as well (corrective maintenance).
- The model assumes the existence of an active resale market where the resale of equipment will attract maximum salvage value.
- It also assumes the existence of an internal market where an in-house use of the machine for less demanding standby capacity will attract a resale value represented by the savings that ensue.
- The resale value is assumed to decrease with deterioration,  $d$  and time in the form given by Lake and Muhlemann [11].

### Features of the Model

- The model is simple and incorporates basic cost components in the build-up of total cost  $K(T)$ .
- It lays emphasis on the effect of deterioration,  $d$  on resale value  $S(t)$  and indeed on the total cost,  $K(T)$ .
- Economic parameters like the rate of return on replacement investment,  $r$  and discount rate,  $R$  are considered in the model. Those costs that occurred at different past points in time need to be discounted to present values to take care of inflation. Replacement investments are forced to compete for capital with alternative investments. Rate of return on replacement investments takes care of the amount of investments on which the company will forgo earning elsewhere. Thus  $R = 100/100 + r$ .
- The model is amenable to simple and straightforward solution techniques like optimization,

dynamic programming and economic lot size inventory control method.

- In the model, deterioration,  $d$  is treated as a stochastic variable and generated as random number under uniform probability distribution. This reduces the error due to the usual subjective methods of determining deterioration.
- The model is amenable to review and adjustments in terms of input variables like deterioration and economic parameters. In fact the model can be restructured if the need arises.
- The model does not involve rigorous enumeration and computation in its solution technique.

**Solution/Results**

The model is calibrated with a set of data from the machines while the verification and comparison are performed with another set of field data obtained from companies earlier mentioned.

**Calibration of the Model**

The new model is also calibrated by the use of a hypothetical machine with the following technical details

- Name of machine: Front-end Loader
- Model: Y
- Year of manufacture: 1995
- Cost: \$60,000.00
- Resale Value (Active Market): \$40,000.00 (5<sup>th</sup> year)
- Deterioration rate: 25%
- Capitalization rate: 40%
- Minor maintenance: \$2,500.00 annually
- Major maintenance:
  - (i) \$10,000.00 (2<sup>nd</sup> year)
  - (ii) \$3,500.00 (3<sup>rd</sup> year)
  - (iii) \$6,000.00 (4<sup>th</sup> year)
  - (iv) \$20,000.00 (5<sup>th</sup> year)

Source: ASA [12]

**Table 1:** This table shows data for model Y, Front-End Loader

t	Q	$b_1$	$b_2t$	S(t)
1	60,000	2500	-	-
2	60,000	2500	10,000	-
3	60,000	2500	3,500	-
4	60,000	2500	6,000	-
5	60,000	2500	20,000	40,000
6	60,000	2500	-	-
7	60,000	2500	-	-

Source: American Society of Appraisers (ASA) [12]

In table 1, column 1 gives the time in years, column 2 gives the cost of the Loader in dollars, column 3 gives the annual routine maintenance costs while column 4 gives the corrective maintenance costs. Column 5 gives the salvage value of the Loader. Enumeration of similar field data for the various machines produces the results in table 3. (See Appendix VI)

**Table 2:** This table shows Random Numbers and Deterioration rates for the Front-end Loader

Random number $X$	Frequency $f$	Probability of X $P(X)$	Deterioration $d$
2	1	0.1	0.33
3	2	0.2	0.50
5	3	0.3	0.83
6	4	0.4	1.00
Total	10	1.0	-

In table 2, column 1 gives the computer generated random numbers for the Loader, column 2 gives the frequency of the random numbers while column 3 gives the probability of occurrence of the deterioration rates in column 4.

Deterioration rates for other machines are obtained in the same manner and are used to derive the salvage values for predicted total costs of the machines.

Table 3: Measured total costs  $K(T)_m$  and predicted total costs  $K(T)_p$  for the Loader, Motor Grader, Capsule filling machine and Injection machine.

In table 3, column 1 gives the time in years, columns 2 and 3 give the measured and predicted total costs for the Loader which reach a minimum at  $t=6$  while columns 4 and 5 give the measured and predicted total costs for the Grader which converge to a minimum at  $t=4$ . Columns 6 and 7 give the measured and predicted total costs for Capsule filling machine which reach a minimum at  $t=16$  for the measured total costs and apparently for the predicted total costs. Columns 8, 9, 10 and 11 give the measured and predicted total costs for the Injection machine. There is no minimum. The Injection machine is in its 4<sup>th</sup> year of service. However, with projected maintenance costs in years 12, 13 and 14, the total costs reach a minimum at  $t=13$ . The total costs of the machines are in Naira except the Loader which is in Dollars. Details of the total costs are presented in Appendix IV (Enumeration Tables)

**Table 3:** This table shows the measured and predicted total costs of the Loader, Grader, Capsule Filling Machine and Injection Moulding Machine

t	Loader		Grader		Capsule Machine		Injection Machine		Injection Machine	
	K(T)m	K(T)p	K(T)m	K(T)p	K(T)m	K(T)p	K(T)m	K(T)p	K(T)m	K(T)p
1	86533	89437	49957100	50082642						
2	96533	99298	50107100	50082642						
3	90033	92888	46969756	46176821						
4	92533	95353	35942700*	37831070*			28787922	28739823	28787922	28760534
5	96136	109158	38763580	46203962						
6	79111*	89435*	41027421	46655690						
7	81234	89435	42841425	47889184						
8	84606	89435								
9					1757946	1736145				
10					2070947	1736147				
11					1761946	1736148				
12					1762946	1736150	38632539	38555951	32632565*	32576681
13					1764446	1736151	32642066	32555978	32642066	32576681*
14					1765946	1736153	32642066	32555980	32642066	32576682
15					2121847	2151155				
16					1718218*	1736156				
17					1727841	1736158				

### Verification of the Model

The model is tested with a set of field data (maintenance costs and salvage values) of machines from Nigerian companies to verify the reliability and operationability of the model.

Maintenance costs and salvage values are obtained for CAT 140H Motor Grader from Mothercat Limited, Kaduna, Nigeria (construction), Leoscar Capsule filling machine from Juhel Nigeria Limited, Enugu, Nigeria (pharmaceutical), Haitian Injection moulding machine from Innoson Technical and Industrial Company Limited, Enugu, Nigeria (Plastics).

### COMPARISON WITH OTHER MODEL

The results of the model on long distance bus and a shuttle bus (Minibus) are compared to the result obtained from the model by Walker [10]. Walker [10] conducted a replacement analysis using data from a fleet of Ford Transit ambulances of a local authority in Northern England. The maintenance costs and salvage values for the Mercedes Benz Express bus and the Toyota Hiace shuttle bus are obtained from Associated Bus Company PLC, Owerri, Nigeria.

The model by Walker [10] is given by

$$R(m) = \left[ P + \sum_{t=1}^m A(t)d^{t-1} - S(m)d^m \right] / \sum_{t=1}^m d^{t-1} \quad 0 \leq m \leq \max$$

Where

- max = a management imposed upper limit on an acceptable replacement cycle.
- R(m) = the equivalent average cost per period for a replacement cycle of  $0 \leq m \leq \max$  periods.
- m = economic life.
- r = rate of return on replacement investment,  $r \geq 0$ .
- d = the discount factor =  $100/(100+r)$ ,  $0 < d \leq 1$
- p = the purchase and installation cost,  $p > 0$ .
- A(t) = the age dependent running cost per period for a machine of age t.  
=  $a_1 + a_2t$ ,  $0 \leq t \leq \max$ .
- a<sub>1</sub> = Constant component of the age dependent running costs (intercept)
- a<sub>2</sub> = increase in age dependent running costs per period (slope)
- S(t) = the resale value of a machine of age t.  
=  $PS_1 (S_2)^t$ ,  $0 \leq S_1 \leq 1$ ,  $0 \leq S_2 \leq 1$ ,  $1 \leq t \leq \max$ .
- S<sub>1</sub> = the deterioration immediately after a purchase (intercept of regression)
- S<sub>2</sub> = the periodic deterioration (slope of regression)

Table 4 gives the measured and predicted total costs for the Express and Shuttle buses as well as the predicted total costs by Walker model (1994) using the cost data of the Express and Shuttle buses. In table 4, column 1 gives the time in years.

**Table 4:** This table shows measured total costs K(T)m and predicted total costs K(T)p for the Express bus, the Hiace bus and the predicted total costs by Walker's model for the Express bus K(T)e and Hiace bus K(T)h.

t	Express K(T)m	Bus K(T)p	Hiace K(T)m	Bus K(T)p	Walker's K(T)e	Value K(T)h
1	31374583	31458812	4879053	4913651	31455741	4913171
2	31379694	31458813	4883053	4913651	31455741	4913171
3	32220387	32289505	5252055	5275652	32286352	5275137
4	31394694	31458813	4892553	4913651	31455741	4913171
5	32207540	33183672	5057162	5466381	33205484	5469610
6	30627980*	31457761*	4577275*	4912726*	31455741	4913172
7	30771993	31458805	4639805	4913651	31455739*	4913171*
8	30907042	31458813	4694926	4913652	31455741	4913172
9			4567649*			

Columns 2 and 3 give the measured and predicted total costs for the Express bus which reach a minimum at  $t=6$ . Columns 4 and 5 give the measured and predicted total costs for the Shuttle bus which also reach a minimum at  $t=6$ . Columns 6 and 7 give the total costs by Walker model (1994) using cost data for the Express and Shuttle buses respectively. Both reach a minimum at  $t=7$ . The total costs are in Naira.

## FINDINGS

- i. Both the measured and predicted total costs, K(T) for the Front end loader reach a minimum at  $t=6$  (Table 3)
- ii. The measured and predicted K(T) for the Loader are positively correlated with correlation coefficient,  $r_c = 0.83$ . The percentage deviation between the measured and predicted total costs for the Loader is low with absolute values ranging from 2.78 to 11.93%.
- iii. The measured and predicted K(T) for the Motor Grader reach a minimum at  $t=4$  (Tables 3).
- iv. The measured and predicted K(T) for the Motor Grader are positively correlated with correlation coefficient  $r_c = 0.81$ . The percentage deviation between the measured and predicted total costs for the Grader is low with absolute values ranging from 0.25 to 16.10%.
- v. The measured K(T) for the Capsule filling machine reaches a minimum at  $t=16$ . (Table 3). The predicted K(T) apparently reaches a minimum also at  $t=16$ .
- vi. The measured and predicted K(T) for the Capsule filling machine are positively correlated with  $r_c = 0.72$ . The percentage deviation between the measured and predicted total costs for the Capsule filling machine is low with absolute values ranging from 0.48 to 19.28%.
- vii. The measured K(T) for the Injection machine does not inflect ordinarily but reaches a minimum at  $t = 13$  with projected maintenance costs (preventive & corrective) in the 12<sup>th</sup>, 13<sup>th</sup> and 14<sup>th</sup> years. The predicted K(T) appears to reach a minimum also at  $t=13$  (Table 3). The Injection machine is in its 4<sup>th</sup> year of operation hence sufficient cost data are yet to be generated for complete enumeration. See attachment for details.
- viii. The measured and predicted K(T) for the Injection machine are highly and positively correlated with  $r_c = 0.99$ . The percentage deviation between the measured and predicted total costs for the Injection machine is low with absolute values ranging from 0.09 to 0.30%.
- ix. The measured and predicted K (T) for the Express bus reaches a minimum at  $t = 6$ . The K(T) of the Walker's model (1994) with data of the Express bus reaches a minimum at  $t = 7$  (Table 4)
- x. The measured and predicted K(T) for the Express bus are positively correlated with  $r_c = 0.81$ . The total costs predicted by both the model and Walker model (1994) are positively correlated with  $r = 0.99$ . Similarly the measured K(T) and predicted K(T) by Walker model for the Express bus are positively correlated with  $r_c = 0.81$ . The percentage deviation between the measured and predicted total costs of the Express bus is low with absolute values ranging from 1.75 to 2.94% for the new model and 1.74 to 3.00% for Walker's model.
- xi. The measured K(T) for the Shuttle bus inflects at  $t = 6$  and 9 with minimum at  $t = 9$ . The predicted K(T)

reaches a minimum at  $t=6$ . The  $K(T)$  of Walker's model with data of the Shuttle bus reaches a minimum at  $t = 7$ . (Table 4)

- xii. The measured and predicted  $K(T)$  for the Shuttle bus are positively correlated with  $r_c = 0.74$ . The total costs predicted by both the model and Walker model (1994) are positively correlated with  $r_c = 0.99$ . Similarly, the measured  $K(T)$  and the predicted  $K(T)$  by Walker model (1994) for the Shuttle bus are positively correlated with  $r_c = 0.74$ . The percentage deviation between the measured and predicted total costs of the Shuttle bus is low with absolute values ranging from 0.43 to 7.49% for the new model and 0.42 to 7.54% for the Walker's model.
- xiii. Both the measured and predicted maintenance costs for the machines are generally non-linear.
- xiv. Maintenance costs (preventive and corrective) increase with time due largely to inflation and deterioration rates.
- xv. Salvage Value (Resale) decreases non-linearly with time due largely to deterioration rate.
- xvi. Generally  $K(T)$  increases with time until it reaches a minimum due to inflation and deterioration rates.
- xvii. As rate of return on investment,  $r$  increases, the discount factor  $R$ , decreases with a corresponding increase in  $K(T)$ .

The maintenance cost function appears to be normally (Log normally) distributed. It may also fit into a gamma distribution since maintenance cost is skewed (higher) towards the end of the useful life of any machine (equipment).

## DISCUSSION

The measured and predicted maintenance costs are generally non-linear [10]. Maintenance costs increase with time due largely to inflation and deterioration rates. Maintenance cost follows a normal (lognormal) distribution and may fit into a gamma distribution since maintenance frequency and costs are higher towards the end of the useful life of any machine.

Total cost increases with time until it reaches a minimum corresponding to the optimal replacement date. Maximum total cost suggests refurbishment. Total cost is influenced by maintenance cost (inflation rate), salvage value (deterioration rate) and insufficient cost data. The remarkable secondhand value of the Grader causes a drop in the value of the measured total cost at  $t = 3$  before reaching a minimum at  $t = 4$ . It also accounts for the variation of the measured and predicted total costs between  $t = 4$  and  $t = 6$ . Generally, total cost increases as

the rate of return on replacement investment,  $r$  increases and discount rate,  $R$  decreases.

The correlation coefficients of the measured and predicted total costs are generally positive and high in value. Thus the measured and predicted total costs in the study are generally dependent with increasing linear relationship. This agreement may be attributed to the quality of the cost data obtained from the field. It is also an indication of the reliability of the new model. The percentage deviation between the measured and predicted total costs of the machines in this study is generally low with absolute values ranging from 0.09 to 19-28 %. This is another clear indication of the quality of the field data as well as the reliability of the new model.

The measured and predicted optimal replacement dates for the machines considered in the study fall within reasonable and acceptable limits with regard to the useful life of the machines. The agreement between the measured and predicted optimal replacement dates is another indication of the reliability of the model.

The measured and predicted results of the study compare favourably with the results obtained by the Walker [9] model in terms of reliability and operation ability as indicated by a correlation coefficient of 0.99. See attachment for the comparison with other models. Assuming regular maintenance and proper operation of the machines, the replacement dates are between 4 and 6 years for construction machines, 16 and 20 years for pharmaceutical machines, 13 and 17 years for plastic and rubber machines, 6 and 7 years for long distance buses, 6 and 9 years for shuttle buses and 6 and 8 years for ambulances. The new model appears to be more suitable for machines with smaller economic life like the construction machines than those with longer economic life like the pharmaceutical equipment. The model is reliable and operational as indicated by its results which compare favourably with those obtained from the model by Walker [9].

## CONCLUSION

The new model is simple, reliable and operational as indicated by high and positive values of the correlation coefficient as well as the agreement between the measured and predicted optimal replacement dates. The model is also amenable to review and adjustment. In fact, parameters like capital allowance, taxation rate and book value of an existing machine may be introduced into the model to enhance its versatility. The model exhibits the characteristics of overlapping sub-problems and optimal substructure [9], hence solution techniques like

optimization, dynamic programming and economic lot size inventory control method are suitable.

The model can address the machinery replacement problems of industries like the construction, pharmaceutical, plastic and transport services provided that there is regular preventive maintenance and timely corrective maintenance. It may also be extended to other industries with the availability of sufficient cost data. However, the model appears to be suitable for machines with small economic life.

The simplicity in the application of the model may lead to savings in terms of cost and time. In fact, the use Monte Carlo simulation to generate values for deterioration under the uniform probability distribution eliminates the cost incurred through failure analysis. The model serves as tool that assists industry managers to make effective machinery replacement decisions since the model predicts machinery replacement dates that reflect real life situation

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## CONFLICT OF INTEREST

No conflict of interests was declared by authors.

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