ISSN 2249-0582 World J of Engineering and Pure and Applied Sci. 2011;1:9
Salawu, 2011. Error Revisited: A Mini Review
OPEN ACCESS

Mini Review

Mathematical Science

Error Revisited: A Mini Review

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ABSTRACT [ENGLISH/ANGLAIS]

This mini review revisits the term error and details its various components in a broad sense. The very complex aspects of error analysis and information regarding philosophy of error have not been included. Interesting and highly functional definitions/descriptions of error are given; the various types of error are explained to some details; some simple methods of minimizing errors are elucidated; and more importantly the applications of error "analysis" in statistical inference (through analysis of variance, ANOVA), in the determination of minimum required sample size, in regression analysis, in categorical data analysis, etc are discussed in some considerably detailed summary. And readers have been referred to some specialized texts wherever appropriate.

Keywords: Error, residual, bias, random, systematic, categorical data analysis, regression

RÉSUMÉ [FRANÇAIS/FRENCH]

Cet avis mini revisite le terme d'erreur et les détails de ses différentes composantes dans un sens large. Les aspects très complexes de l'analyse des erreurs et la philosophie des informations concernant d'erreur n'ont pas été inclus. Intéressant et très fonctionnel définitions / descriptions d'erreur sont donnés; les différents types d'erreur sont expliqués à certains détails, certaines des méthodes simples de réduire les erreurs sont élucidés, et surtout les demandes d'erreur "l'analyse" en inférence statistique (par l'analyse de la variance , ANOVA), dans la détermination de la taille minimale de l'échantillon nécessaire, dans l'analyse de régression, l'analyse des données catégoriques, etc sont examinés en résumé considérablement détaillée. Et les lecteurs ont été renvoyées à des textes spécialisés le cas échéant.

Mots-clés: Erreur, résiduelle, de partialité, aléatoire, systématique, l'analyse des données catégoriques, la régression,

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Accepted/Accepté: May, 2011

Citation: Salawu EO. Error Revisited: A Mini Review. World Journal of Engineering and Pure and Applied Sciences 2011; 1:9-14.

INTRODUCTION

In this mini review the term "error" is carefully discussed in a broad scene. However, this mini review is limited to the simple scientific discussion of the term (error) and so we do not proceed into the very complex analyses of errors and the derivation of any very complex equations. Also, we do not view error in the philosophical sense, since these are not the target of this mini review. We advise readers who are interested in some specialized aspects of error and analyses of errors (such as error analysis in numerical modeling, error analysis in molecular dynamics simulation, etc) to consult some other available resources such as Taylor [1], Bevington [2], Barford [3], Beers and Robinson [4], Brach [5], and many others.

MEANING OF ERROR

The meaning of error mainly depends on the context. Going back in time one would find that "the concrete meaning of the Latin word 'error' is 'wandering' or 'straying'" [6]. For the sake of this review (and since, in this review error is being given a fisheye approach) one defines error simply as a state of straying from the truth which essentially makes one to be different/deviate from the state of being accurate (or perfect). However, one should be clear at this point that error and mistake are essentially different. In the actual sense, we may say that a mistake is an error, but the truth is that all errors are not mistakes. For example, even though one may carefully perform experiments (say simple measurements) and avoid all possible forms of human fault or carelessness (that is mistake) one is always bound to have some errors in experiments and the best one could do is minimize the error in some ways (which are discussed latter). One could carefully say that while mistakes can be avoided in experiments, somehow, errors cannot.

With the simple (and yet, highly functional) descriptions of error presented here, one can easily proceed and discuss the various "types" of errors, the various applications of error, and so on.

TYPES OF ERROR

We broadly classify errors in two types: (1) Random Error; and (2) Systematic Error. Just as the names imply, Random Errors result from some random variations which are essentially in both directions and about the average/central point. On the other hand, systematic errors result from some forms of non-random, constant, and one-sided deviation from the average/central point. It is also worth of note that, while there are standardized methods for quantifying random errors, such methods do not really exist for systematic errors.

Random Error

Random errors are errors which merely result by chance. They should be seen as random variations which could not really be accounted for by any covariate (under study, or otherwise). With large number of observations, random errors are approximately normally distributed about the central point. With increase in the number of observations (or sample size) towards infinity, the distribution of standardized values of random error approaches standard normal with mean equal to zero, and standard deviation equal to one (unity).

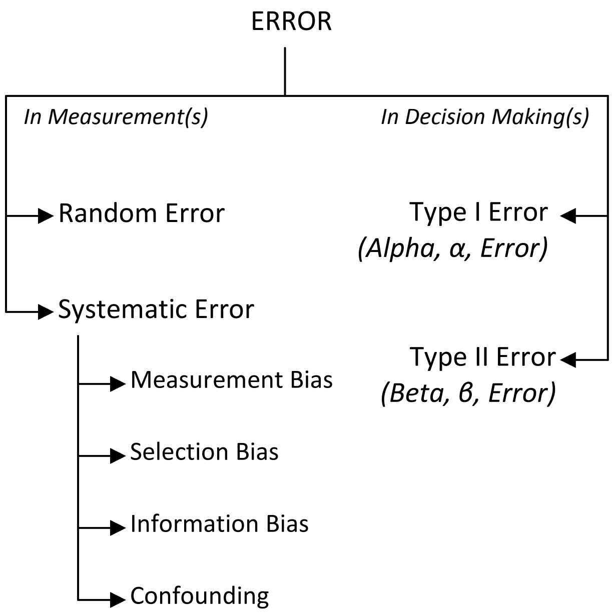
Systematic Error

In truth systematic errors are not due to chance(s), rather they can be assigned to some (systematic) causes. For such reasons, systematic errors tend to be consistent in one direction and thus push the observation towards a single side. This makes systematic error to be a differential error, and to also be called a bias. The presence of systematic errors can be accounted for by covariate(s) [for example, by confounder(s) such as in observational studies], negligence (for example, not setting measuring instrument to the zero point prior to measurement, or using a faulty instrument) or even by some behavioral/intentional preferences and other various types of biases.

Even though, more than thirty different forms have been clearly identified, one could broadly classify systematic errors (also known as biases) into four main types (and further sub-classify each of the four types as appropriate). The four main types are Measurement Bias, Selection Bias, Information Bias, and Confounding.

FIGURE 1

Figure 1 shows Schematic Representation of Categorization of Errors



While it is very reasonable to say that errors are undesirable and should always be minimized, it is also very important for one to understand the various aspects of science (and various other subjects) where the applications of knowledge of errors are indispensable. But before proceeding to the demerits, minimization, and the applications of the knowledge of error (analysis), readers may find it interesting to know the difference between a type I error and a type II error (which may be seen as another way of classifying errors).

Type I and Type II Errors

While Random Error and Systematic Error are the classifications of error that has found greatest attention in measurements, Type I and Type II Errors classification has found more attention in decision/choice making. To easily explain (and for readers to vividly remember) each of Type I and Type II Errors and distinguish between then, a 2 x 2 contingency table would be an indispensable tool.

Suppose a test is used in distinguishing between good and bad (or say, between positive and negative), one knows that the four possible combinations can be adequately represented in a 2 x 2 contingency table [7], such as the one shown below.

TABLE 1

Table 1 shows a 2 x 2 contingency table for explaining and also making simple differentiation between Type I and Type II Errors

		Condition (i.e. Actual Situation)	
		+	-
Test	+	Correct Decision	Type I Error
	-	Type II Error	Correct Decision

One must understand that type I error is the error committed when a test shows positive (that is, the test confirms the presence of a condition) when in the actual fact it should show a "negative" (that is, the absence of the condition). This is a false positivity, and it is a very serious error that many (in fact, all) scientists consider to be highly unacceptable, and thus they always fix the probability (*p*) associated with it to some very value (often below 5%). The consciousness of the possibility of this type I error is of high application in inferential statistic, where statisticians (as much as other researchers) always desire that the probability (*p*) of committing this error be very small (and approach zero) when they attempt to reject the null hypothesis of no difference.

On the other hand, type II error is committed when a test fails to detect the presence of a condition when, in the actual fact, the condition is present. Reasonably, one says that type II error is "false negativity." Interestingly, type II error is never seen to be such a great scientific crime like type I error, and the main problem with type II error is that (if all other conditions are kept constant) the higher the type II error the lower the power of a test. But this is also undesirable.

Unlike in type I error, where sample size may be irrelevant, increasing the sample size does reduce the probability of committing type II error. In fact, one way of increasing the power of a test, which is equivalent to reducing the probability of type II error (while fixing the probability of type I error) is increasing the sample size.

Readers who are interested in some further analysis/discussion of both type I and type II errors are advised to check the next issue of this journal where it is hoped that a well detailed calculations/analyses/discussions of these types of error would be published. Alternatively, such readers may find many other scholarly articles (e.g. Lemons et al. [8]) and books that have carefully and considerably covered this and similar topics through the internet.

DEMERITS, MINIMIZATION, AND "MERITS" OF ERRORS

While errors are generally viewed in the negative sense, scholars (especially scientists) who understand that errors (especially random errors) are somehow unavoidable have learnt to "live" with it and in fact take advantage of it (wherever possible).

Demerits of Errors

The most important problems with errors center around the inaccuracy, imprecision, loss in power of tests, and reduced efficiency they cause in measurements. Since random errors are divergence from the central/true points due to chance and are essentially random variations on the either side of the central point, their main problems are associated with reduced precision in measurements rather than gross inaccuracy of measurements. When an experiment involves attempts to establish a link between two or more variables (say through correlation, odds ratio, relative risk, regression models, etc), random error weakens the strength of association; and considerably reduces the power of tests in inferential statistics. As for systematic error, the situation is often worst since the systematic error tends to be one-sided and is non-random. With this one-sided nature of deviation from the true point, in systematic error, many scholars have said that systematic error is in fact the same as bias. The greatest evil in systematic error is that instead of merely weakening the association (such as in random error), it is capable of considerably distorting the nature of the association and in many cases, systematic error may revert the direction of the association and may create great confusion and misdirection. In such cases completely wrong conclusions might be reached and Type I error might result.

Some Simple Methods of Minimizing Errors

Random Error

Simply put, one can at the long run reduce the magnitude of the marginal effects of random error by increasing sample size or increasing the number of replicates (as appropriate). But is it worthy of note that completely eradicating random error is just impossible.

Systematic Error

Knowing that systematic error is a very broad term referring to a very broad category of all differential errors, one would be aware of the fact that various many ways are available for the minimization/eradication of systematic errors.

Even though, it is highly reasonable (although somehow traditional, according to some people) to believe that randomization is a universal way of reducing a number of systematic errors, still the most appropriate way of preventing, minimizing, or eradicating a particular type of systematic error depends mainly on the nature of the error as well as the context. For example, while blinding may be appropriate for controlling some types of bias (a typical systematic error), correctly setting the zero point would be the appropriate way of minimizing some other systematic errors, and carefully controlling the effects of the third (unstudied) variable could be the best way of controlling confounding, and so on. Even though random errors can not be completely eradicated (but can only be reduced), virtually all forms of systematic errors can be completely removed if appropriate measures are taken.

Merits of (Knowledge of) Errors

Readers should not get carried away by the heading of this subsection: Merits of Error. In fact, it is very hard to say that errors are important, and this would be more of philosophy than pure and/or applied science. Notwithstanding, the very important concept of precision which we all find to be essential in measurements/experiment has its bases in the clear understanding of errors and, in fact, mathematically one could say that precision is inversely proportional to error; which means that precision goes down as error increases. Thus, the subheading (Merits of Error) has merely been selected so as to draw the attention of readers to the very important fact that the knowledge of (or about) errors is of great importance in virtually all fields of study, and especially in mathematical sciences and engineering, medicine, business and commercial studies, and in defense and criminology. In the later part of this review, the applications of the knowledge of error (and some of the simple analysis regarding error) in mathematical sciences and engineering (with special reference to statistics) are briefly discussed.

SOME VERY SIMPLE APPLICATIONS OF KNOWLEDGE OF ERRORS

Determination of Sample Size

Of high importance is the application of error (often quoted as acceptable level of error, precision, absolute precision, relative precision, etc) in the estimation of minimum required sample size in both observational and experimental studies. A good understanding of the maximum acceptable level of error (often denoted by d, l, e, D, L, or E) is, in fact, highly essential in the development of research studies (be it observational or experimental). This is mainly because there is undisputable (and mathematically proven) relationship between sample size, variance, precision (and error), power (of a test), and reliability and validity (or representativeness) of the outcome of a study. A researcher (from almost any field whatsoever) would, therefore, always want to be sure of the minimum (and sometimes optimum) required sample size for his research before proceeding. This estimation of the minimum (or optimum) required sample size would not be possible without (a least) some knowledge of error. Even for health studies Lwanga and Lemeshow [9] have published a Practical Manual for sample size determination in health studies, and the authors considerably recognized the importance of acceptable level of error (presented as either absolute or relative error) in the estimation of minimum required sample size.

Generally we say that the minimum required sample size for a study/research is inversely proportional to the inverse of the acceptable level of error (which could be fixed by the research, but with reference to some previous knowledge or some essential phenomena).

Analysis of Variance

Another very great application of the information about error is found in the analysis of variance (commonly called ANOVA), which is, in reality, one of the most widely used statistics techniques in various fields of study.

ANOVA recognizes the fact that the estimation of the significance of the effect of a factor/treatment/treatment combination (say, the effect of varying temperature on efficiency of a printing machine) depends on contrasting the variations due to the factor/treatment/treatment combination with the variation due to error. For example, this is simply done by dividing the mean sum of squares due to the factor/treatment/treatment combination by the mean sum of sum of squares due to error. By this, one obtains a factor/multiplier which depicts the number of times the factor/treatment/treatment combination has more effects on the outcome variable than does the random error. If and only if the factor/treatment/treatment combination has far more effect on the outcome than does the error, than one concludes (through the proper use of F distribution) that the noted effect of factor/treatment/treatment combination on the outcome is truly assignable to factor/treatment/treatment combination and is not likely to have resulted from error/chance.

In essence, what we do in ANOVA (and many other tests/analyses involving variance) is that we quantify the mean variation that is due to error/chance and the mean variation that is due to the factor/treatment/treatment combination of interest and we find their ratio (with the former being the denominator, and the latter being the numerator), and finally we base the decision of whether the obtained value (from the ratio) is larger than the tabulated values of F (in F distribution table) at the appropriate numerator and denominator degrees of freedom or not. In fact, analysis of variance is not possible without knowing the variations due to error as well as the degrees of freedom of error.

Analysis of Residuals

It is what of note that errors can be seen as residuals, which can be simply defined as the difference between expected value (say, mean) of an observation and the observed value. Information regarding residuals has found a lot of applications in many mathematical/statistical sciences, but here we briefly consider regression analysis and categorical data analysis.

Analysis of Residuals in Regression Analysis

Linear regression modeling (and many other reductions of generalized linear model, GNL) deems incomplete without proper analysis of what we often call residuals – the differences between observed and estimated.

Especially in linear regression modeling, it has been categorically stated by many scholars (for example, by Chatterjee and Price [10] in their book "Regression analysis by Example") that a high index fit (e.g. multiple correlation coefficient, R^2) and highly significant regression coefficients (beta's) do not necessarily mean that a model has been well and fitted. If truth be told, analyses of residuals (say, through residual plots) often reveal a lot of model violations if present; and those model violations a not uncommon even when R^2 is large and model coefficients are highly significant.

It is interesting to know that even though there are some few other methods for the detection of some model violations, a number of them do not produce results that are as reliable as the results that analyses of residuals produce. For example, Durbin-Watson statistics that is used for the detection of autocorrelation works fine only for first order autocorrelation, whereas a careful use of residual analysis/plot would, if present, always reveal any form of autocorrelation whatsoever.

Other applications of residual plots/analyses in regression analysis are detection of outliers, detection of heteroscedasticity, detection of multicolinearity, and the suggestion of the appropriate measures for the correction of these problems, to name a few. More comprehensive and easy to understand information regarding residual analysis could be found in Regression Analysis by Example [10].

Analysis of Adjusted Residuals in Categorical Data Analysis

While Pearson's (X^2) statistic, maximum likelihood statistics (G^2), and test statistics (M^2) of dependence in ordinal data, can show/explain the nature of the relationship between two categorical variables, they are limited in that they do not provide the cell-to-cell information regarding the targeted dependency. This brought about the necessity for the method of adjusted residuals. In this method, adjusted residuals are computed for each of the cells in the cross-tabulated levels of the categorical variables being studied.

Whenever the magnitude of the adjusted residuals is larger than 2 (i.e. 2 SD's, equivalent of 95% confidence level), one says that cells being compared are significantly different and that the difference is due to the level/categories of the second categorical variable. Here one is able to understand the cell-to-cell dependency in the categories of the two variables. Such detailed cell-to-cell information would not have been obtainable if one sticks rigidly to the ordinary Pearson Chi-Squared Statistics or even the Maximum Likelihood Statistics, and we thus concludes that the use of adjusted residuals provide vital information on the cell-to-cell behavior of the dependency between variables being studied. In fact, it would be reasonable to say that Cochran-Mantel-

Haenszel method of analysis of dependency (testing for the type of dependency and the estimation of pooled odds ratio) in 2 x 2 x k tables also have some bases which are intimately linked to residuals. Reader who are interested in further information regarding calculations and the use of adjusted residuals in categorical data analysis and/or the details of Cochran-Mantel-Haenszel method are directed to textbook of categorical data analysis [11].

CONCLUSION

This mini-review concludes with a charge that scholars and researchers in all fields of study should familiarize need to familiarize themselves with (at least) some basic concepts regarding types, demerits, and minimization of errors, as well as with the applications of these concepts in their field of study, be it in the designing of their research studies, in the analysis of data or in the interpretation of obtained results.

REFERENCES

[1] Taylor JR. An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements. 2nd ed. California: University Science Books; 1997.
[2] Bevington PR, Robinson DK. Data Reduction and Error Analysis for the Physical Sciences. 3rd ed. Boston: McGraw-Hill Higher Education; 2003.
[3] Barford NC. Experimental Measurements: Precision, Error, and Truth 2nd ed. New York: John Wiley & Sons Inc; 1985.
[4] Beers Y. Introduction to the Theory of Error. Reading, MA: Addison-Wesley; 1957.
[5] Brach, R., "Uncertainty in Accident Reconstruction Calculations," SAE Technical Paper 940722, 1994.
[6] Wikipedi 2011. Accessed from: http://en.wikipedia.org/wiki/Error Last Accessed: May 2011.
[7] Salawu EO. Degrading Standard of Examinations in Many Higher Institutions. World Journal of Life Sciences and Medical Research 2011;1:1-4.
[8] Lemons J, Shrader-Frechette K, Cranor C. The Precautionary Principle: Scientific Uncertainty and Type I and Type II Errors. Foundations of Science 1997;2(2):207-36.
[9] Lwanga KS, Lemeshow S. Sample size determination in health studies. A practical manual. Geneva: World Health Organization; 1991.
[10] Chatterjee S, Price B. Regression analysis by example. New York: John Wiley & Sons Inc; 1977.
[11] Agresti A. Categorical Data Analysis. 2nd ed. New Jersey, Hoboken: John Wiley & Sons, Inc.; 2002.

ACKNOWLEDGEMENT / SOURCE OF SUPPORT

Nil

CONFLICT OF INTEREST

No conflict of interest was declared by authors.